Spectrum-matching Techniques for Shallow-water Remote Sensing

Lecture 3A: Semianalytical Models

Curtis Mobley

Vice President for Science and Senior Scientist
Sequoia Scientific, Inc.
Bellevue, WA 98005
curtis.mobley@sequoiasci.com

IOCCG Course
Villefranche-sur-Mer, France
July 2014

Spectrum-matching Algorithms

We are going to use radiometrically calibrated and atmospherically corrected R_{rs} spectra to simultaneously retrieve bottom depth, bottom reflectance (bottom type), and water column absorption and scattering properties via “spectrum matching” to the full spectra.

Two basic types of “radiative-transfer-based” algorithms for spectrum matching:

- **Semianalytical:** Start with radiative transfer theory and derive an approximate analytical model relating R_{rs} to bottom depth, reflectance, etc. Then use the image R_{rs} determine best-fit values for the parameters of the model via nonlinear optimization (Lee et al, 1998,1999, Applied Optics)
- **Database Search:** First use a radiative transfer code to create a database of R_{rs} spectra that correspond to all possible combinations of water absorption and scattering properties, bottom depths, and bottom reflectances that might be found in the area being studied. Then match image spectra to the database spectra. (Mobley et al., 2005, Applied Optics)

Semianalytical Models

These models seek to improve upon statistical models by building in known (e.g., derived from observation or the RTE) relations between the inputs and outputs.

“Analytical” means derived (with a pencil and paper) using various approximations, simplifications, or assumptions. The math model still has various unknown parameters, which must be determined before the model can be used to invert R_{rs} to obtain what is wanted.

The unknown parameters are determined by forcing the model to fit data containing both the inputs (R_{rs}) and outputs (Chl, water depth, etc), hence “semi” analytical: part analytical and part statistical.

After the parameters have been determined using known inputs and outputs, the model with the same parameter values can be applied to new input data, to obtain new outputs.

Generic Procedure for Semianalytical Models

- Model R_{rs} = f(bottom depth, bottom reflectance, water IOPs, sun angle, etc.)

- Quasi-single-scattering approximation (QSSA) shows that for deep water,

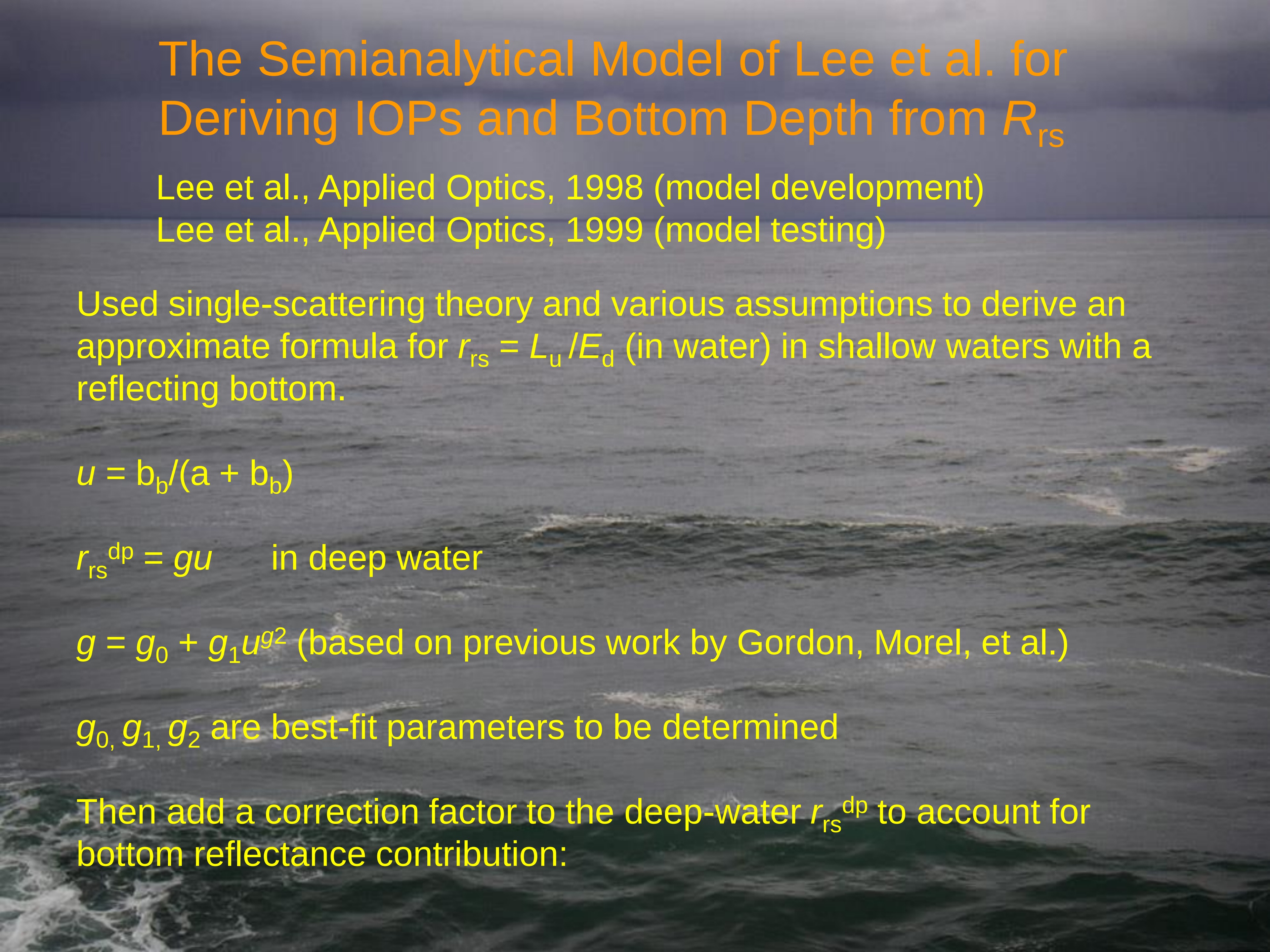
  $R_{rs} \sim b_b/(a+b_b)$

www.oceanopticsbook.info/view/radiative_transfer_theory/level_2/the_quasisinglescattering_approximation

- Use analytic functions for a and b_b components
  - $a = a_w(\lambda) + A_1 a_1^*(\lambda) + A_2 a_2^*(\lambda) + \ldots + A_m a_m^*(\lambda)$
  - $b_b = b_{bw}(\lambda) + B_1 b_{b1}^*(\lambda) + B_2 b_{b2}^*(\lambda) + \ldots + B_n b_{bn}^*(\lambda)$
  - A_m and B_n are eigenvalues (magnitudes; to be determined)
  - $a_m^*(\lambda)$ and $b_{bn}^*(\lambda)$ are eigenvectors (spectral shapes; assumed known)

- We assume reasonable shapes for the eigenvectors of different water-column components (phytoplankton, CDOM, etc.).

- Solve for eigenvalues via non-linear least-squares minimization (or some other technique) when fitting the model R_{rs} to the measured R_{rs}



The Semianalytical Model of Lee et al. for Deriving IOPs and Bottom Depth from R_{rs}

Lee et al., Applied Optics, 1998 (model development)
Lee et al., Applied Optics, 1999 (model testing)

Used single-scattering theory and various assumptions to derive an approximate formula for $r_{rs} = L_u / E_d$ (in water) in shallow waters with a reflecting bottom.

$u = b_b/(a + b_b)$

$r_{rs}^{dp} = gu$ in deep water

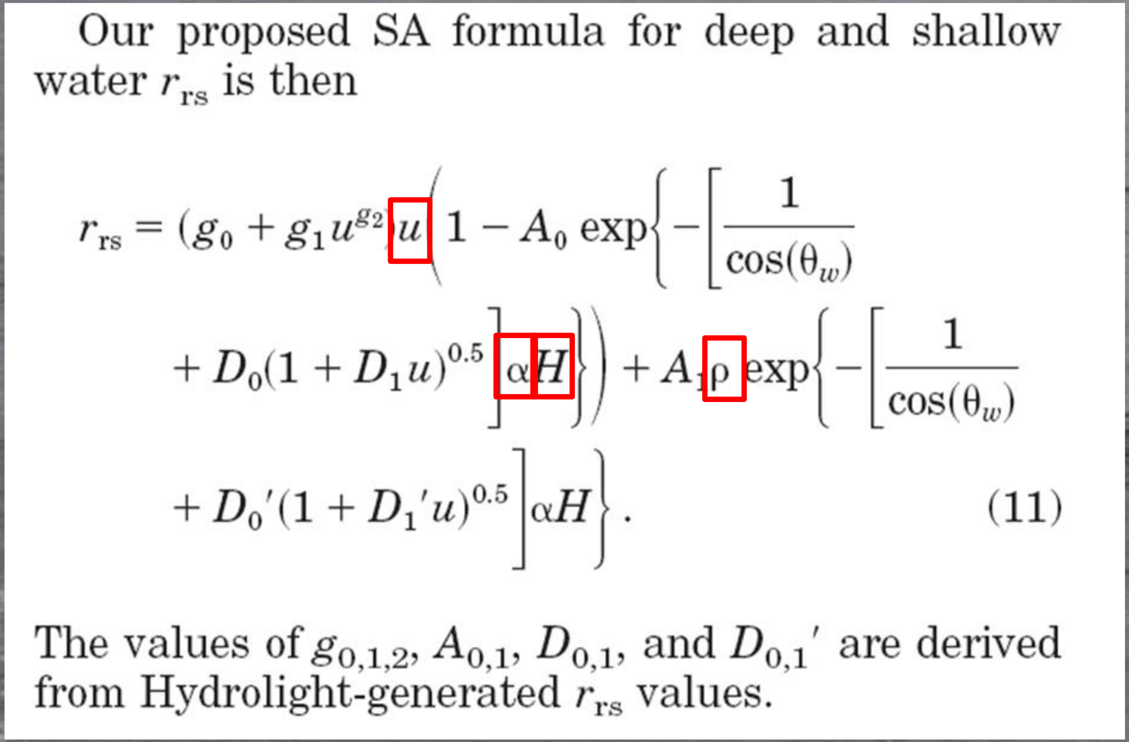
$g = g_0 + g_1 u^{g2}$ (based on previous work by Gordon, Morel, et al.)

g_0, g_1, g_2 are best-fit parameters to be determined

Then add a correction factor to the deep-water r_{rs}^{dp} to account for bottom reflectance contribution:

$r_{rs} = r_{rs}^{dp}$ {1 - A_0 exp [-(K_d + K_u^C) *H*] } + A_1 ρ exp [-(K_d + K_u^B) *H*]

QSSA: $K_d \sim (a + b_b)/\cos(\theta_{sun, water})$, so rewrite $K_d = D_d(a + b_b)$, etc. to get



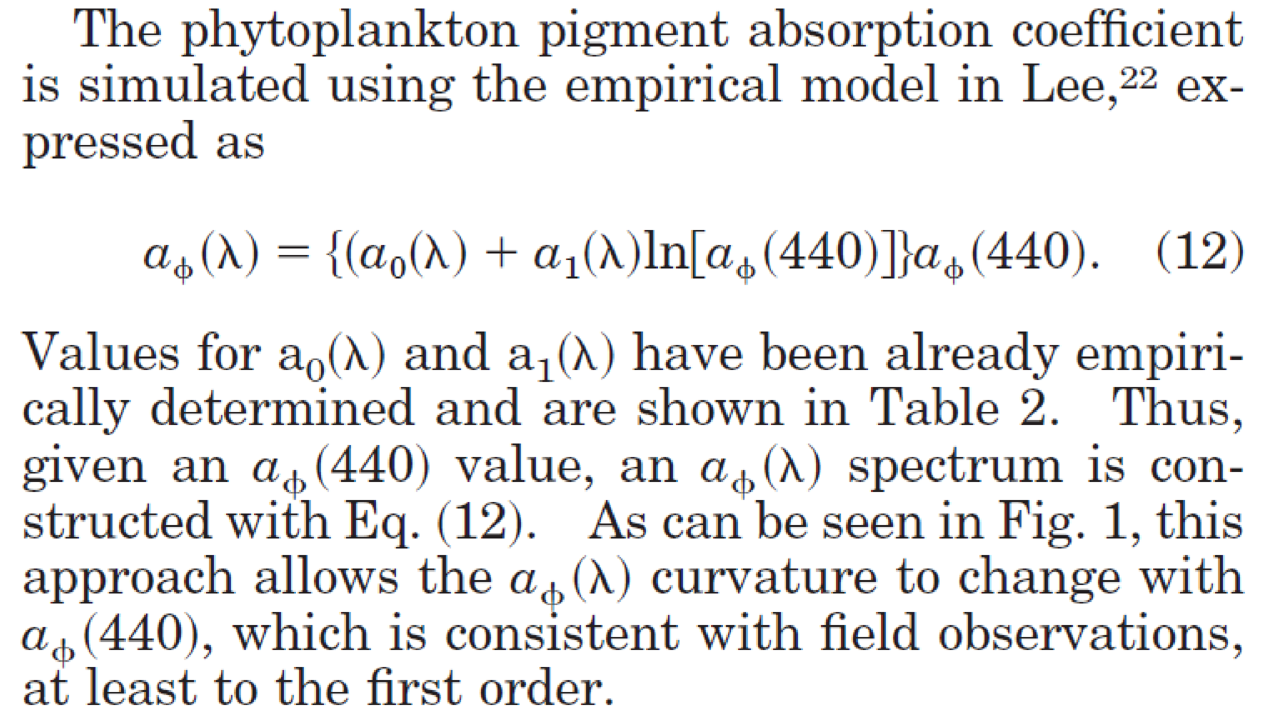
Our proposed SA formula for deep and shallow water r_{rs} is then

$$r_{rs} = (g_0 + g_1 u^{g_2}\boxed{u})\left(1 - A_0 \exp\left\{-\left[\frac{1}{\cos(\theta_w)} + D_0(1 + D_1 u)^{0.5}\right]\boxed{\alpha}\boxed{H}\right\}\right) + A_1\boxed{\rho}\exp\left\{-\left[\frac{1}{\cos(\theta_w)} + D_0'(1 + D_1'u)^{0.5}\right]\alpha H\right\}. \qquad (11)$$

The values of $g_{0,1,2}$, $A_{0,1}$, $D_{0,1}$, and $D_{0,1}'$ are derived from Hydrolight-generated r_{rs} values.

$\alpha = a + b_b$, ρ is the bottom reflectance, θ_w is the in-water sun zenith angle, and *H* is the bottom depth

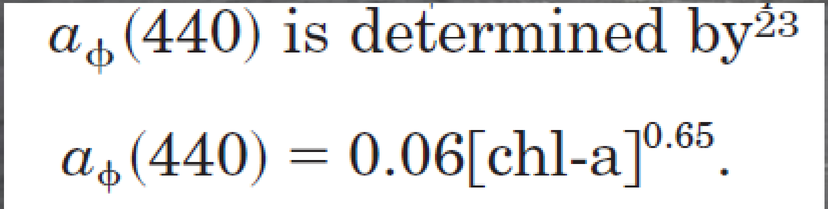
Spectral dependence of *a* and b_b via simple functions



The phytoplankton pigment absorption coefficient is simulated using the empirical model in Lee,[22] expressed as

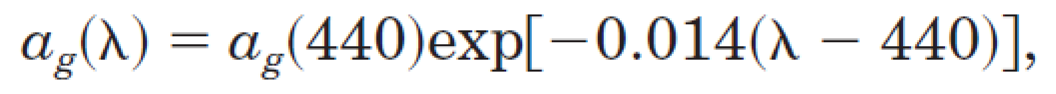
$$a_\phi(\lambda) = \{(a_0(\lambda) + a_1(\lambda)\ln[a_\phi(440)]\}a_\phi(440). \quad (12)$$

Values for $a_0(\lambda)$ and $a_1(\lambda)$ have been already empirically determined and are shown in Table 2. Thus, given an $a_\phi(440)$ value, an $a_\phi(\lambda)$ spectrum is constructed with Eq. (12). As can be seen in Fig. 1, this approach allows the $a_\phi(\lambda)$ curvature to change with $a_\phi(440)$, which is consistent with field observations, at least to the first order.

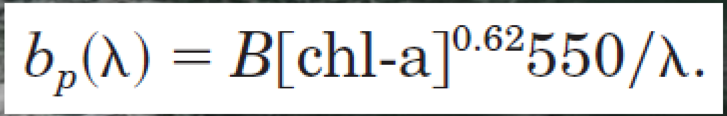


$a_\phi(440)$ is determined by[23]

$$a_\phi(440) = 0.06[\text{chl-a}]^{0.65}.$$

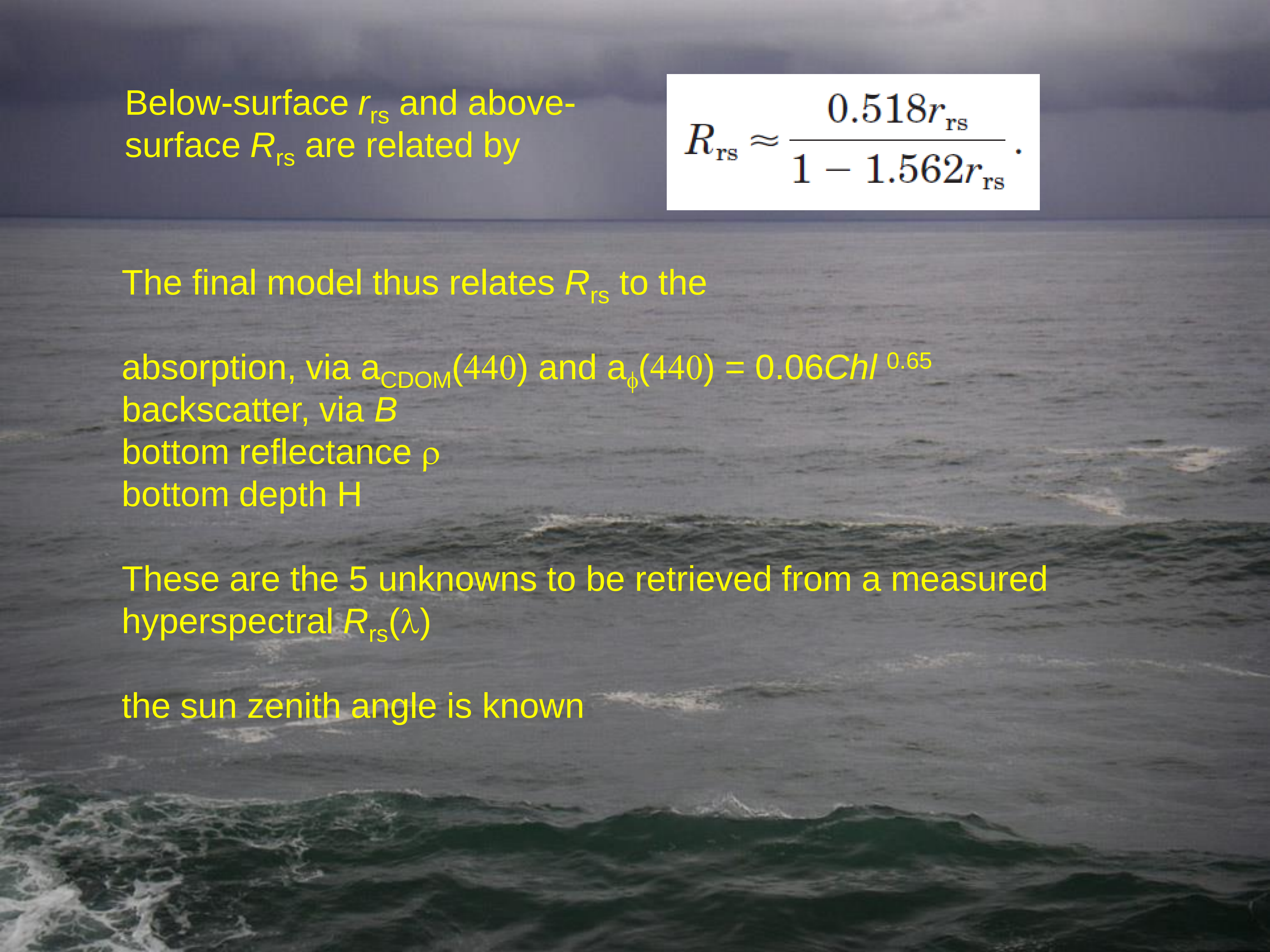


$$a_g(\lambda) = a_g(440)\exp[-0.014(\lambda - 440)],$$



$$b_p(\lambda) = B[\text{chl-a}]^{0.62}550/\lambda.$$

water column IOPs are modeled in terms of just 3 numbers: $a_\phi(440)$ or Chl, $a_g(440)$, and *B*



Below-surface r_{rs} and above-surface R_{rs} are related by

$$R_{\mathrm{rs}} \approx \frac{0.518 r_{\mathrm{rs}}}{1 - 1.562 r_{\mathrm{rs}}} .$$

The final model thus relates R_{rs} to the

absorption, via $a_{CDOM}(440)$ and $a_{\phi}(440) = 0.06 Chl^{0.65}$
backscatter, via B
bottom reflectance ρ
bottom depth H

These are the 5 unknowns to be retrieved from a measured hyperspectral $R_{rs}(\lambda)$

the sun zenith angle is known

Then used HydroLight for a wide range of input IOPs, bottom depths, sun angles, etc. to generate r_{rs} spectra for known $a_{CDOM}(440)$, $a_{\phi}(440)$, B, ρ, and H, which were then fit with the model to determine values of the model parameters $g_{0,1,2}$, $A_{1,2}$, $D_{1,2}$, $D'_{1,2}$

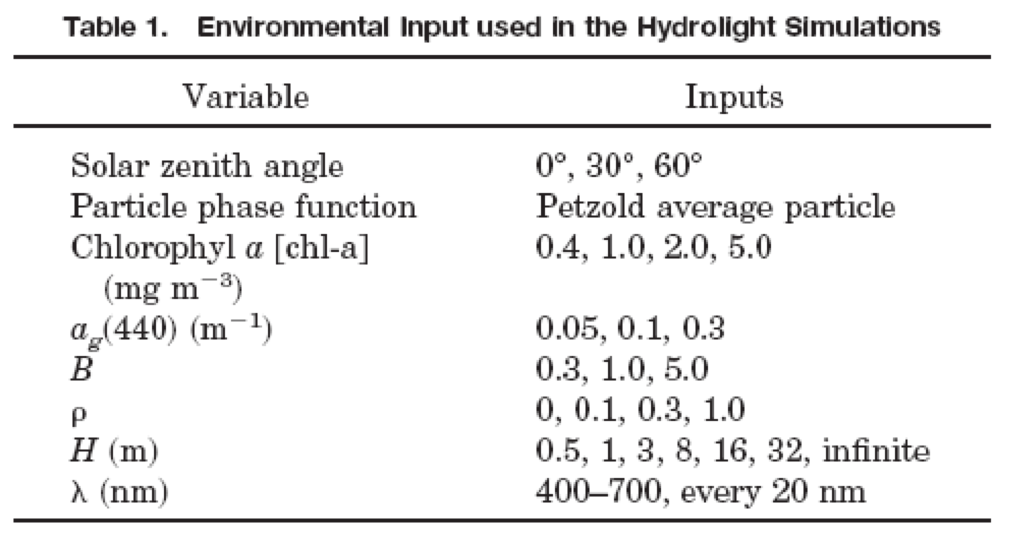
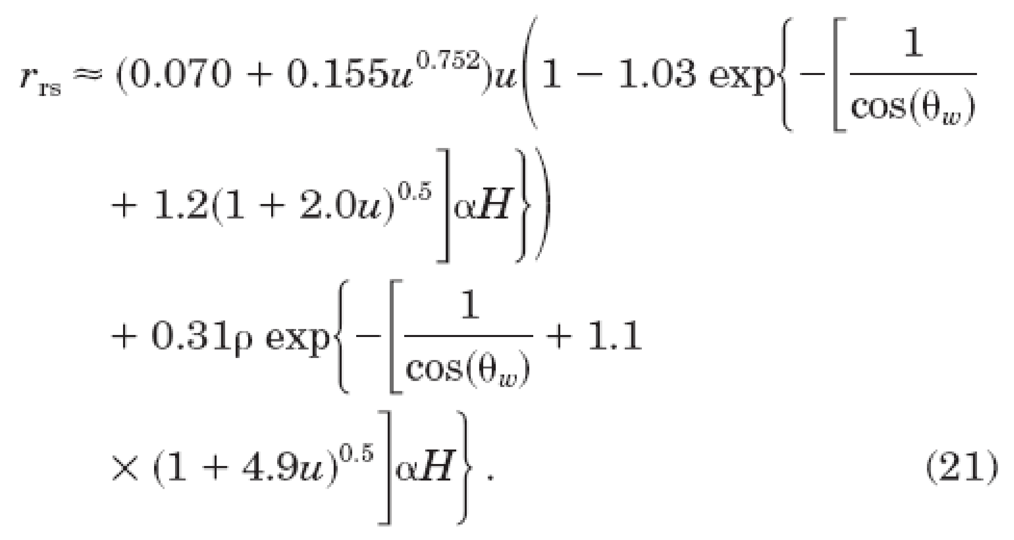


Table 1. Environmental Input used in the Hydrolight Simulations

Variable	Inputs
Solar zenith angle	0°, 30°, 60°
Particle phase function	Petzold average particle
Chlorophyl *a* [chl-a] (mg m^{-3})	0.4, 1.0, 2.0, 5.0
$a_g(440)$ (m^{-1})	0.05, 0.1, 0.3
B	0.3, 1.0, 5.0
ρ	0, 0.1, 0.3, 1.0
H (m)	0.5, 1, 3, 8, 16, 32, infinite
λ (nm)	400–700, every 20 nm

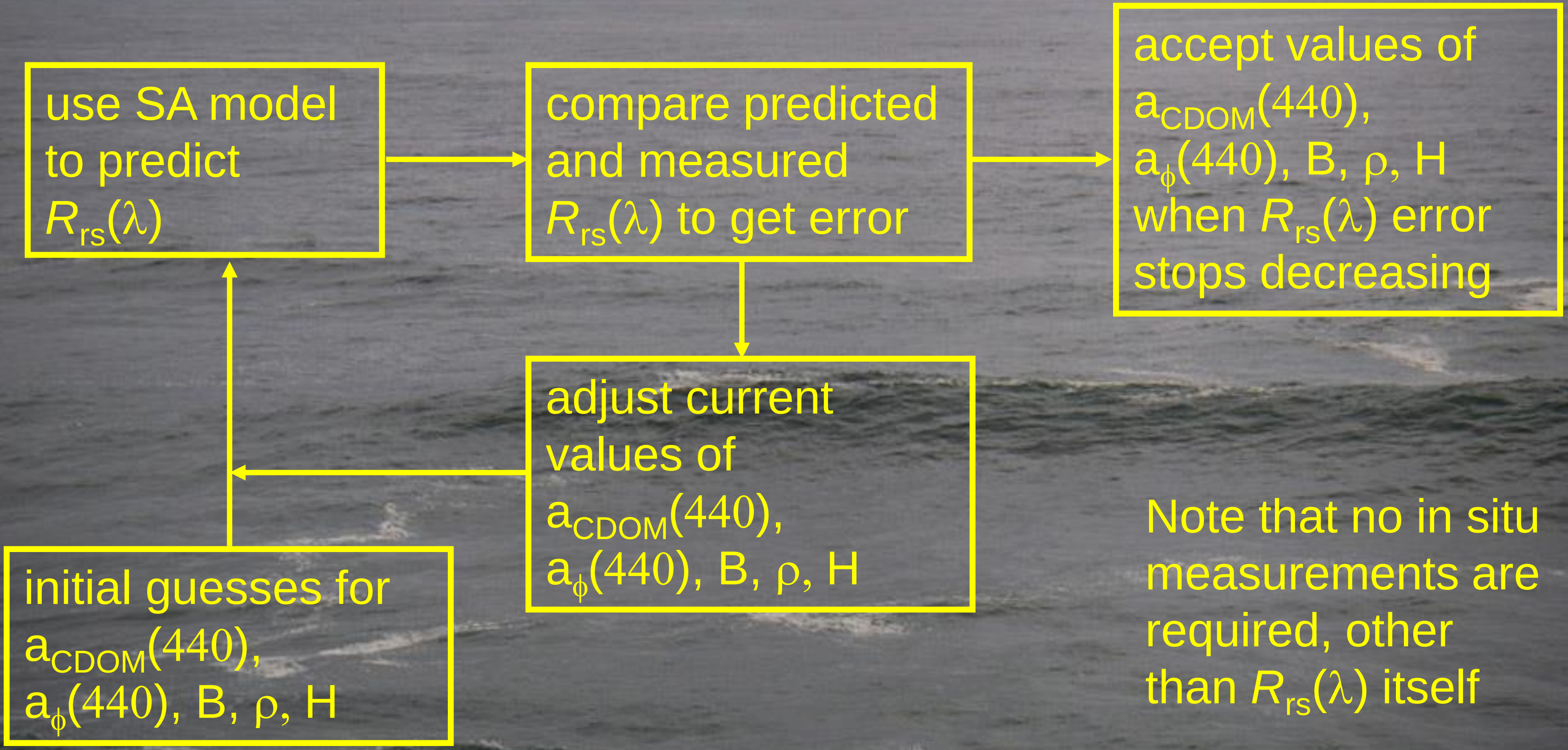
used HydroLight to generate pseudo data for determining parameter values because no real data were available



$$r_{rs} \approx (0.070 + 0.155u^{0.752})u\left(1 - 1.03\exp\left\{-\left[\frac{1}{\cos(\theta_w)} + 1.2(1 + 2.0u)^{0.5}\right]\alpha H\right\}\right) + 0.31\rho\exp\left\{-\left[\frac{1}{\cos(\theta_w)} + 1.1 \times (1 + 4.9u)^{0.5}\right]\alpha H\right\}. \qquad (21)$$

The final model with best-fit parameter values shown

The final model is then fit to a measured hyperspectral $R_{rs}(\lambda)$ spectrum using a predictor-corrector algorithm to retrieve IOPs and depth



initial guesses for $a_{CDOM}(440)$, $a_{\phi}(440)$, B, ρ, H

use SA model to predict $R_{rs}(\lambda)$

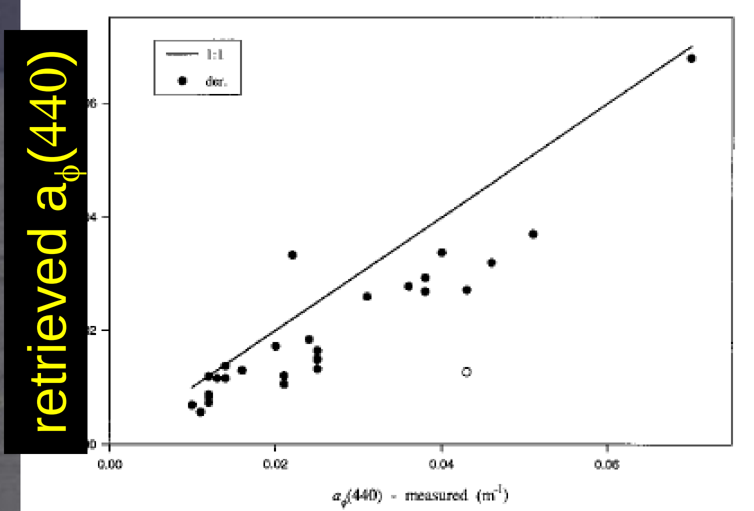
compare predicted and measured $R_{rs}(\lambda)$ to get error

accept values of $a_{CDOM}(440)$, $a_{\phi}(440)$, B, ρ, H when $R_{rs}(\lambda)$ error stops decreasing

adjust current values of $a_{CDOM}(440)$, $a_{\phi}(440)$, B, ρ, H

Note that no in situ measurements are required, other than $R_{rs}(\lambda)$ itself

Model Validation:
Retrieved vs Measured Phytoplankton Absorption



measured $a_\phi(440)$

Fig. 13. Inve … ent at 440 nm compared … (The open circle one was … error calculation.)

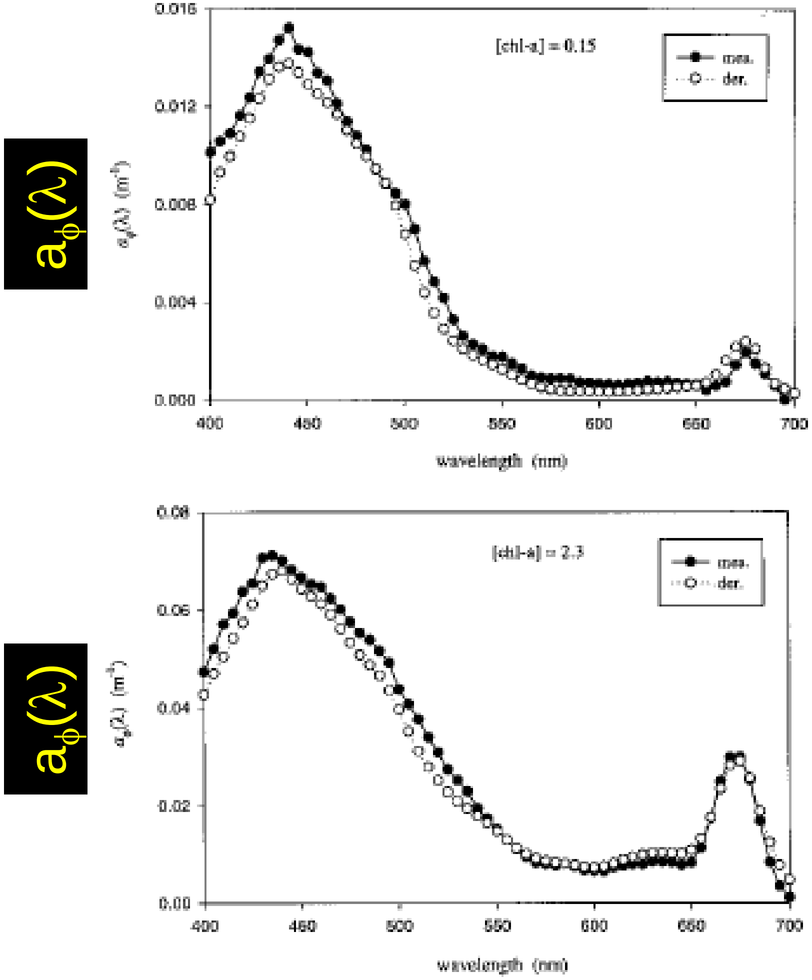


Fig. 14. Examples of inversion-derived versus pad-measured pigment absorption spectra.

Model Validation:
Retrieved vs Measured CDOM Absorption and Depth

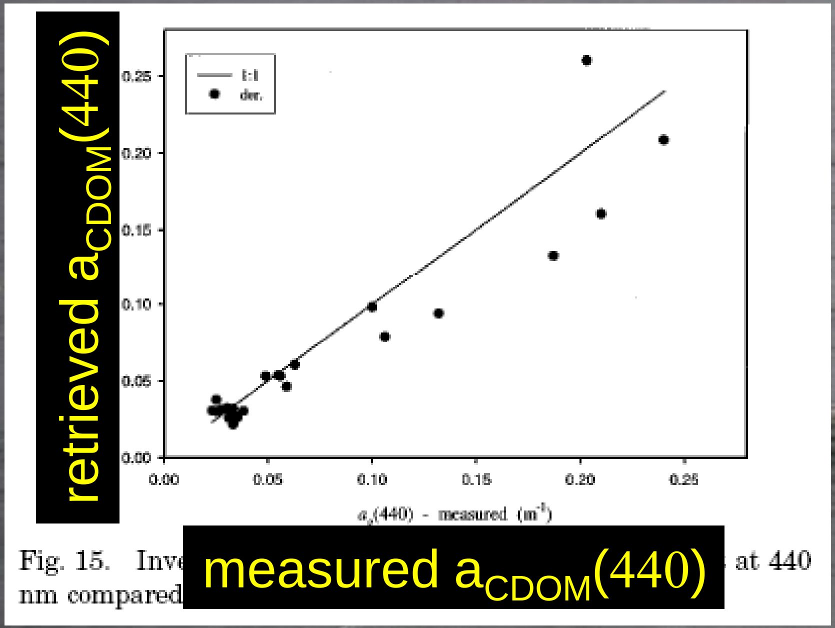


Fig. 15. Inve … at 440 nm compared

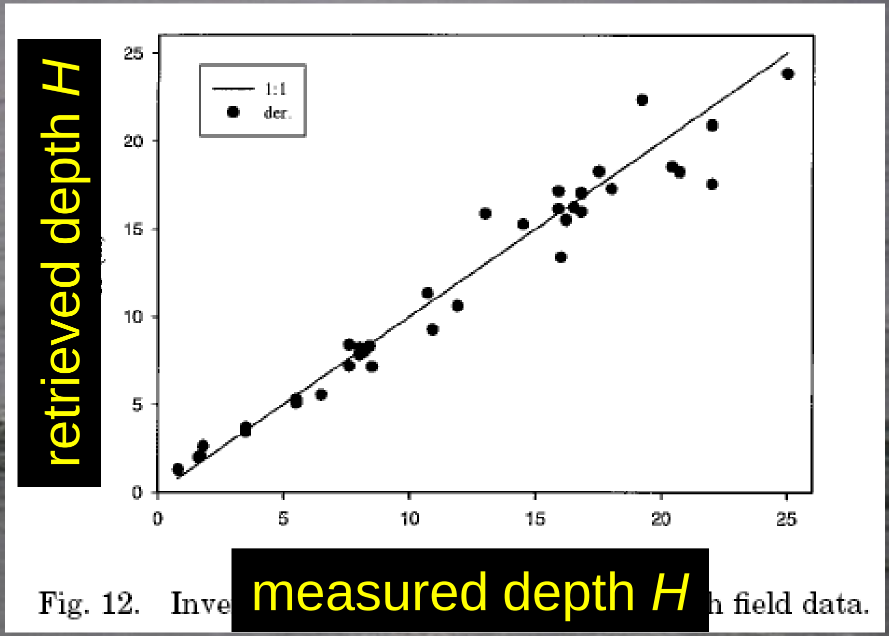


Fig. 12. Inve … h field data.

Later Work

Building on the seminal work of Lee et al., a number of other semi-analytical models for shallow-water retrievals have been developed:

HOPE: Hyperspectral Optimization Process Exemplar (the Lee model)

BRUCE: Bottom Reflectance Unmixing Computation of the Environment (Klonowski et al., 2007; modfied the bottom reflectance parameterization to include mixture of 3 bottom types)

SAMBUCA: Semi-Analytical Model for Bathymetry, Unmixing, and Concentration Assessment (Brando et al., 2009; added NAP to water and modified bottom reflectance paramerization)

ALLUT: Adaptive Linearized Look-Up Trees (Hedley et al., 2009; combines features of SA model and database search described next)

See Dekker et al (2011, L&O Methods) for details, references, and comparsions.

Summary

- Semi-analytic reflectance inversion models are powerful tools for estimating bottom depth, bottom reflectance, and spectral IOPs from ocean color imagery (the Lee model is used for HICO retrievals)

- The devil is in the details…
  - basis vector definitions
  - solution methods: non-linear regression, optimized non-linear regression, linearized regression, etc.
  - initial guesses; if not good, iterative solution may not converge or solution may be unphysical

- important considerations
  - testing against independent measured observations
  - sensitivity analysis (e.g., to different phase functions)
  - uncertainties in assumptions used to derive the model

Kayaking Doesn't Get Any Better Than East Greenland