Lecture 1a
Overview of Radiometry

Curtis Mobley

Vice President for Science
Senior Scientist
Sequoia Scientific, Inc.
Bellevue, Washington 98005 USA
curtis.mobley@sequoiasci.com

IOCCG Course
Villefranche-sur-Mer, France
July 2016

Who Am I?

B.S. in Physics (1969, Univ Texas at Austin)
Ph.D. in Meteorology (1977, Univ Maryland at College Park)
(with other studies at Univ. Fridericiana, Karlsruhe, Germany and the Univ. of International Relations, Beijing, China)

40 year career in optical oceanography, mostly doing radiative transfer numerical modeling and remote sensing

Author of *Light and Water: Radiative Transfer in Natural Waters*

Developed the HydroLight radiative transfer software

Recent work in airborne remote sensing of optically shallow water, ecosystem modeling, and polarization

Along the way: University of Washington; Program Manager for the US Navy; various research labs and companies; V.P. for Science at Sequoia Scientific, Inc. since 1996.

Resources

- *Light and Water*:   Distributed on CD. OK for basic theory and math, but now out of date in some sections.

- www.oceanopticsbook.info   This website site is intended to be continually under development and never complete; community inputs are welcome.

- pdfs of papers and of lectures

- HydroLight User's Guide and Technical Documentation; hardcopy and on the HydroLight/Documents directory

- ASK ME QUESTIONS. That is why I am here! We have time for questions and discussions (complicated philosophical questions may be best answered over a glass of wine at dinner). Ask me to slow down when I talk too fast.

The Grand Scheme

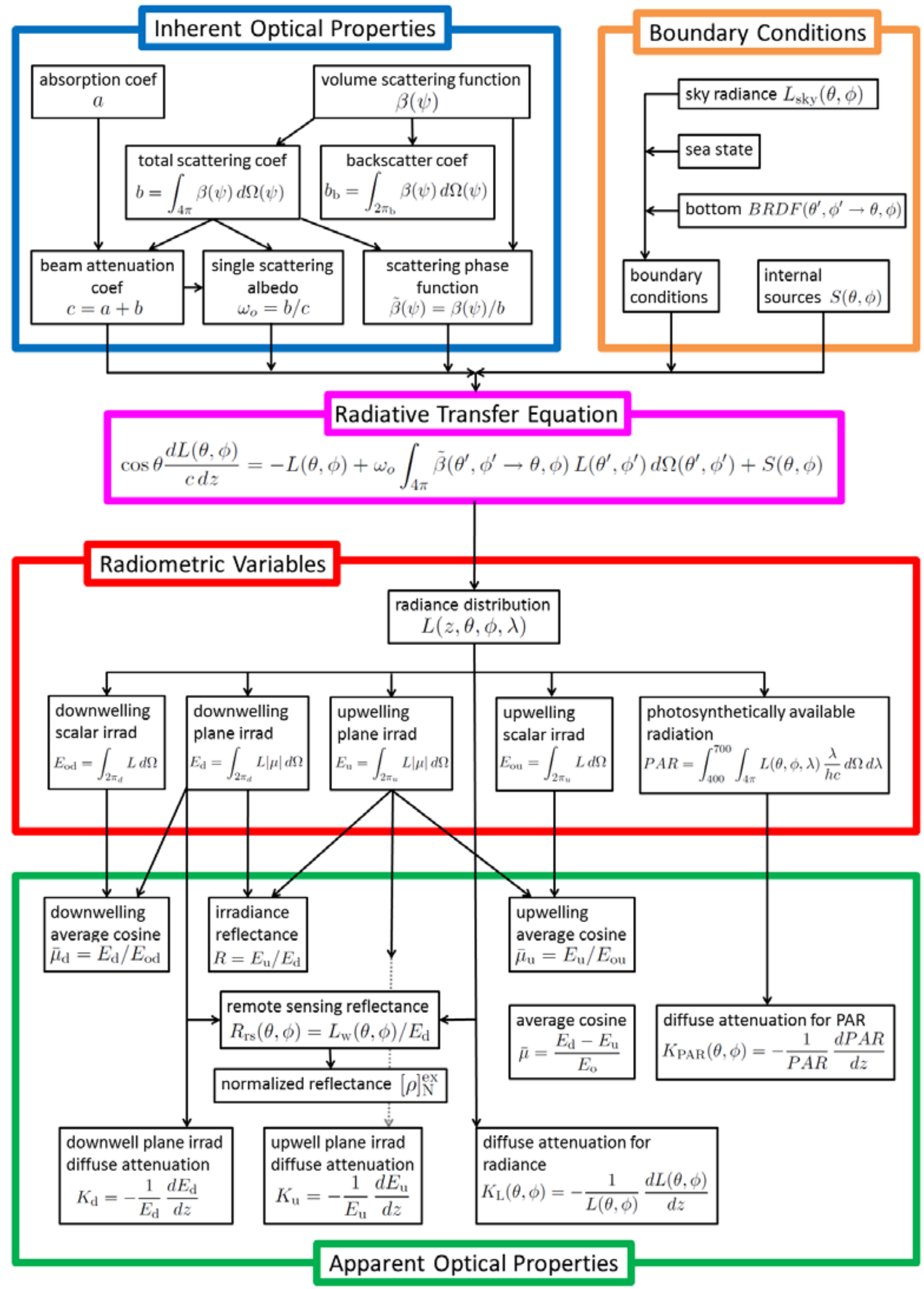
Inherent Optical Properties (IOPs) define the absorption and scattering properties of the water

Boundary conditions at the sea surface and bottom are needed to solve the RTE

The Radiative Transfer Equation (RTE) relates the IOPs (the water) to the radiometric variables (the light)

The radiometric variables (radiance and irradiances) describe the light

The Apparent Optical Properties (AOPs) depend both on the IOPs and the radiance distribution



Light and Radiometry

"The beginning of wisdom is to call things by their proper names"
-- Confucius

Basic knowledge needed for this course:

What is light? How do you describe how much light there is, where it is going, etc.?

How to specify directions

Radiance—the fundamental quantity for describing light

Irradiances—easier to measure than radiance and often more useful

What is Light?

Light consists of elementary particles called *photons*, which are characterized by their

- *wavelength* λ (or frequency ν)
- state of *polarization*

Photons

- carry *energy* and linear and angular *momentum*
- have *no* detectable *physical size* (point particles, like electrons??)
- always travel at the *same speed in a vacuum*

Both wave and particle properties are always present (can't be described by classical physics). However, you can *use either* the wave *or* the particle properties to describe light, depending on which is convenient for your particular problem (e.g., photons are created (emitted) or destroyed (absorbed) like particles, but they propagate or scatter like waves). You will measure *either* the wave or particle properties of light, depending on the *measurement device* being used in the experiment.

Like electrons, no one knows what photons "really are," but their behavior can be predicted. That's all that matters ("shut up and calculate").
(see Griffiths, *Intro to Elementary Particles* for the ideas of QED)

Example Calculations

How much energy does one photon have?

How many photons are there in visible light on a typical day at sea level?

Energy q of a photon is $q = h\nu = \frac{hc}{\lambda}$

where

- ν is frequency [1/sec]
- λ is wavelength (in a vacuum), [meters]
- $h = 6.626 \cdot 10^{-34}$ J s is Planck's constant
- $c = 2.998 \cdot 10^{8}$ m s^{-1} is speed of light (in vacuo)

and also

$$\lambda_{\text{medium}} = \lambda \frac{c_{\text{medium}}}{c} = \frac{\lambda}{n}$$

or

$$n = \frac{c}{c_{\text{medium}}}$$ is the index of refraction

How many photons per m^2 at sea level?

Light and Water, Table 1.4, typical day: 400 Wm^{-2}

so for $\lambda = 550 \cdot 10^{-9}$ m (green light)

$$\frac{\frac{400 \text{ J}}{\text{s m}^2}}{\frac{(6.6 \cdot 10^{-34} \text{ J s})(3 \cdot 10^{8} \text{ m s}^{-1})}{550 \cdot 10^{-9} \text{ m}}} \approx 10^{21} \frac{\text{photons}}{\text{m}^2 \text{ s}}$$

How many photons per m^3 at near the surface?

$$\frac{\frac{10^{21} \text{ photons}}{\text{m}^2 \text{ s}}}{\frac{3 \cdot 10^{8} \text{ m s}^{-1}}{1.34}} \approx 4 \cdot 10^{12} \frac{\text{photons}}{\text{m}^3} \approx \frac{\text{number of phytoplankton}}{\text{m}^3}$$

but each phytoplankton is getting "hit" by many photons per second

How to specify directions

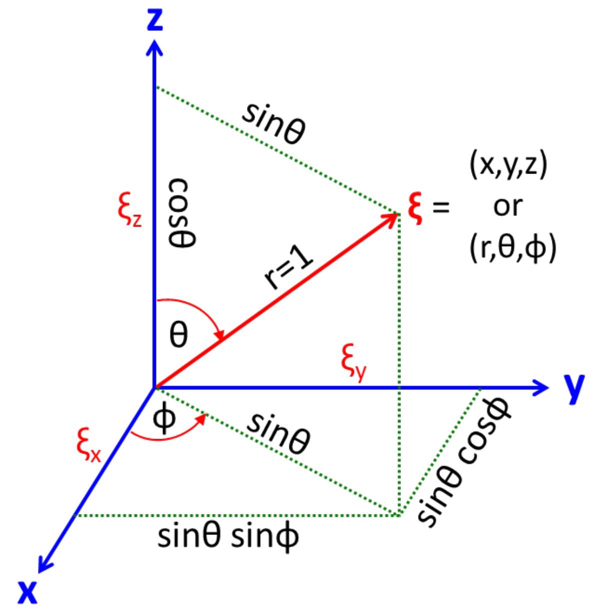
unit vectors for direction

Warning on directions:

In radiative transfer theory (i.e., in the radiative transfer equation, in *Light and Water*, and in HydroLight), θ and ϕ always refer to the direction the light is going.

Experimentalists often let θ and ϕ refer to the direction the instrument was pointed to measure the radiance.

I call the instrument direction the viewing direction, θ_v and ϕ_v, where $\theta_v = \pi - \theta$ and $\phi_v = \phi + \pi$.



$\hat{\xi}$ is a *unit vector* specifying direction (θ, ϕ), so

$$|\hat{\xi}| = 1 = \hat{\xi} \cdot \hat{\xi} = \xi_x^2 + \xi_y^2 + \xi_z^2$$

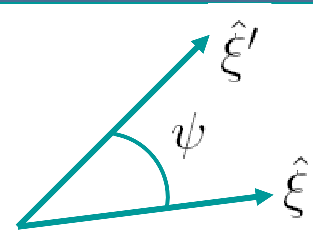
$$\begin{aligned} \hat{\xi} &= \xi_x \hat{x} + \xi_y \hat{y} + \xi_z \hat{z} \\ &= (\sin\theta\cos\phi)\hat{x} + (\sin\theta\sin\phi)\hat{y} + (\cos\theta)\hat{z} \end{aligned}$$

$$\theta = \cos^{-1}(\xi_z) \qquad \mu \equiv \cos\theta$$

$$\phi = \tan^{-1}\left(\frac{\xi_y}{\xi_x}\right)$$

For more detail, see *Light and Water*, Chapter 1 and www.oceanopticsbook.info/view/light_and_radiometry/geometry

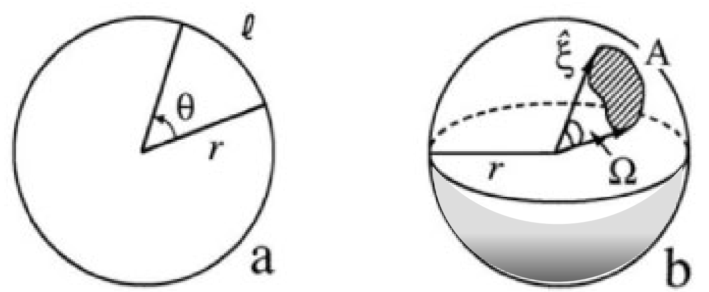
Computing the (scattering) angle between two directions



$$\hat{\xi}' \cdot \hat{\xi} \equiv |\,\hat{\xi}'\,||\,\hat{\xi}\,|\cos\psi = \cos\psi.$$

$$\begin{aligned}\cos\psi &= \xi'_x\xi_x + \xi'_y\xi_y + \xi'_z\xi_z \\ &= \cos\theta'\cos\theta + \sin\theta'\sin\theta\cos(\phi'-\phi) \\ &= \mu'\mu + \sqrt{1-\mu'^2}\sqrt{1-\mu^2}\cos(\phi'-\phi)\end{aligned}$$

Defining solid angles

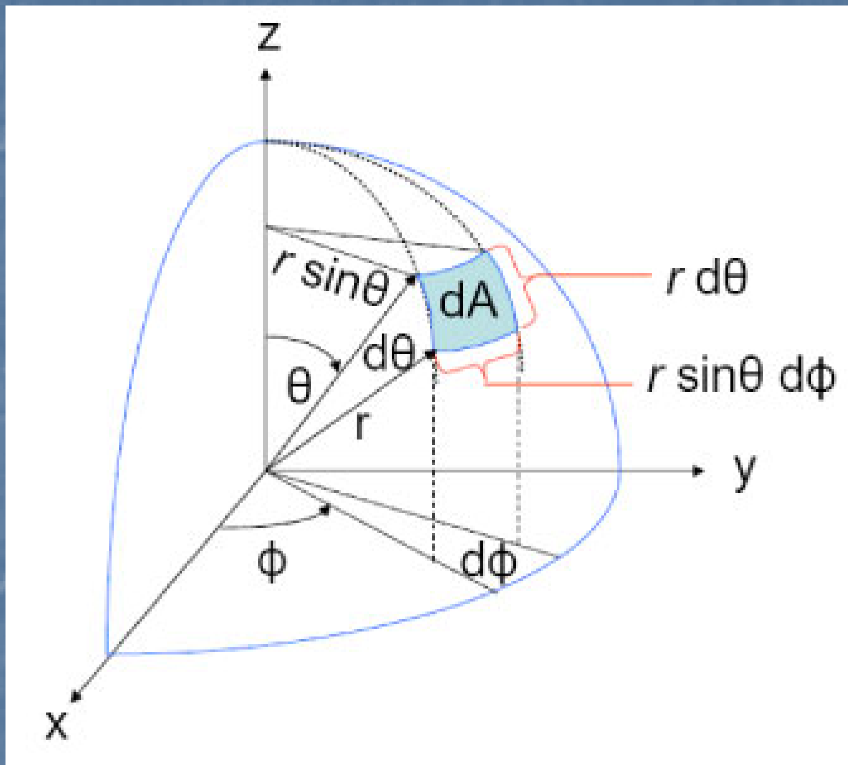


$$\text{angle} \equiv \frac{\text{arc length}}{\text{radius}} \qquad \text{solid angle} \equiv \frac{\text{area}}{\text{radius squared}}$$

$$\theta = \frac{\ell}{r} \quad \text{(radian)} \qquad \Omega = \frac{A}{r^2} \quad \text{(steradian)}$$

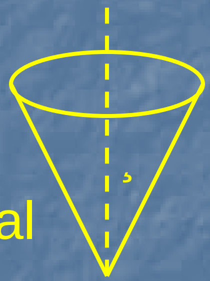
$$\text{circle} = 2\pi \text{ rad} \qquad \text{sphere} = 4\pi \text{ sr}$$

Computing Solid Angles



$$d\Omega = \frac{dA}{r^2} = \frac{(r sin\theta\, d\phi)(r d\theta)}{r^2} = \sin\theta\, d\theta\, d\phi$$
$$= d\mu d\phi$$

Example: What is the solid angle of a cone with half-angle ¸ ?



Place the cone pointing to the "north pole" of a spherical coordinate system.

$$\Omega = \int_{\phi'=0}^{2\pi} \int_{\theta'=0}^{\theta} \sin\theta' d\theta' d\phi' = 2\pi(1-\cos\theta)$$
$$= \int_{\phi'=0}^{2\pi} \int_{\mu'=\mu}^{1} d\mu' d\phi' = 2\pi(1-\mu)$$

Radiometry

Radiometry is the science of measuring electromagnetic (radiant) energy

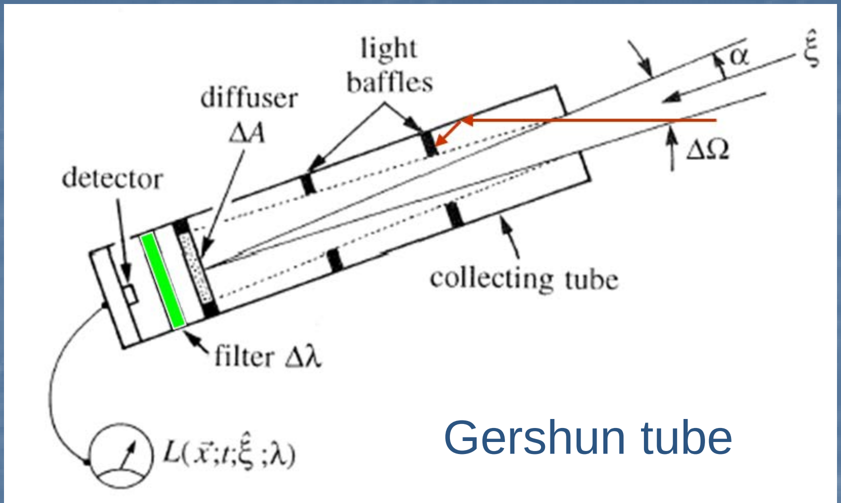
Two types of detectors:

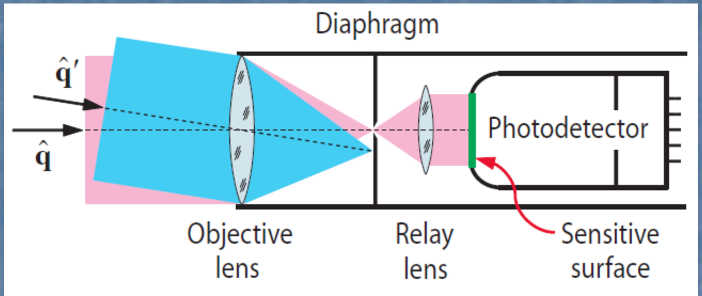
• thermal—instrument response is proportional to the *energy* (absorbed and converted to heat)

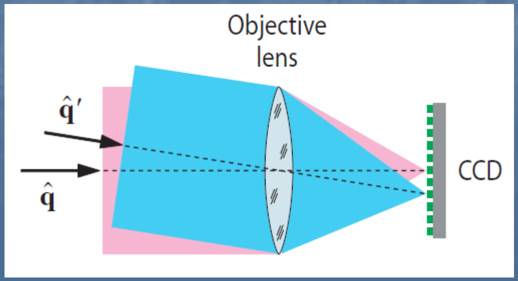
• quantum—instrument response is proportional to the *number of photons*

Calibration of radiometric instruments is very difficult (~2% accuracy at best)

Radiometers







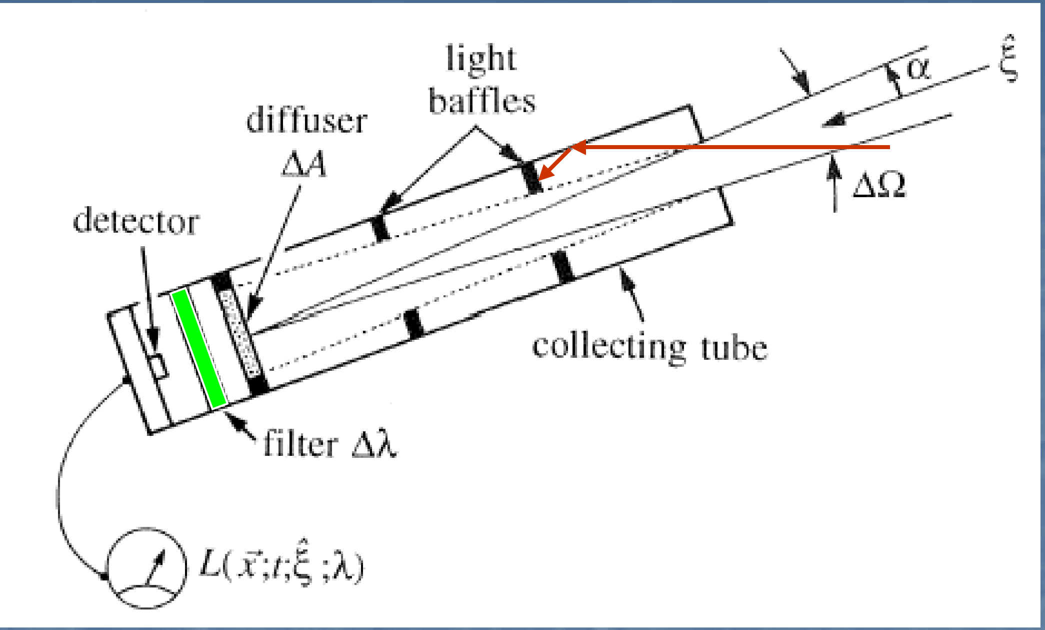
from Mishchenko (2014)

Spectral Radiance

If you know the radiance, you know everything there is to know about the light field

Full specification of the radiance at a given location and time includes its state of polarization, wavelength, and direction

"spectral" can mean either "per unit of wavelength or frequency" or "as a function of wavelength"



$$L(\vec{x}, t, \hat{\xi}, \lambda) \equiv \frac{\Delta Q}{\Delta t \, \Delta A \, \Delta\Omega \, \Delta\lambda}$$

$$(\mathrm{J\ s^{-1}\ m^{-2}\ sr^{-1}\ nm^{-1}})$$

$$(\mathrm{W\ m^{-2}\ sr^{-1}\ nm^{-1}})$$

HydroLight computes *L*(*z*,¸ ,Æ;»)

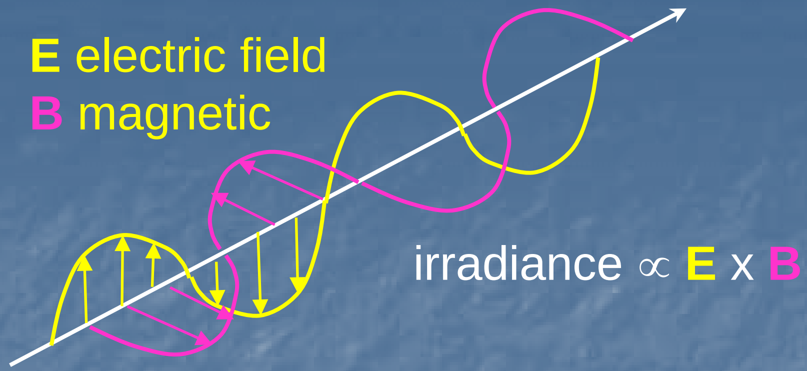
Polarization

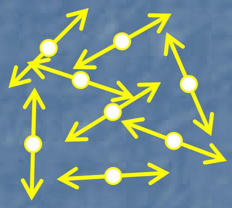
The polarization state of the radiance requires four numbers: the elements of the Stokes vector $S = [I, Q, U, V]^T$

I gives the total radiance, without regard for the state of polarizaiton. Q, U, V describe the linear and circular polarization of the light.

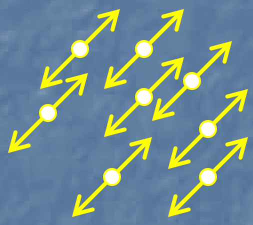
$S = L(z, \theta, \phi, \lambda)\ [1, 0, 0, 0]^T$ represents unpolarized radiance

$S = L(z, \theta, \phi, \lambda)\ [1, 0.3, 0, 0]^T$ shows that the total radiance is partially linearly polarized

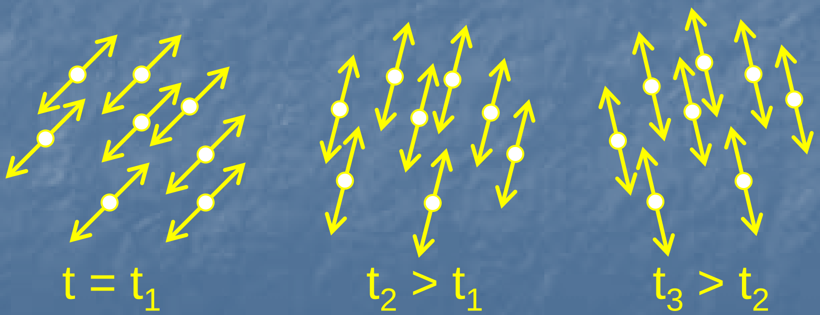




randomly polarized (unpolarized):
E fields are randomly oriented



plane polarized:
E fields are all in the same plane (constant in time)



circularly polarized:
E fields rotate with time

The state of polarization of the radiance contains information about the environment (such as the size distribution, shape, and index of refraction of particles in the water)

However, oceanographers usually measure only the total radiance because

- The 4x1 Stokes vector (and corresponding 4x4 Mueller matrix, which describes scattering of polarized light) is much harder to measure than just total radiance

- The state of polarization is believed to have little effect on processes like phytoplankton photosynthesis or water heating (which depend on absorbed energy)

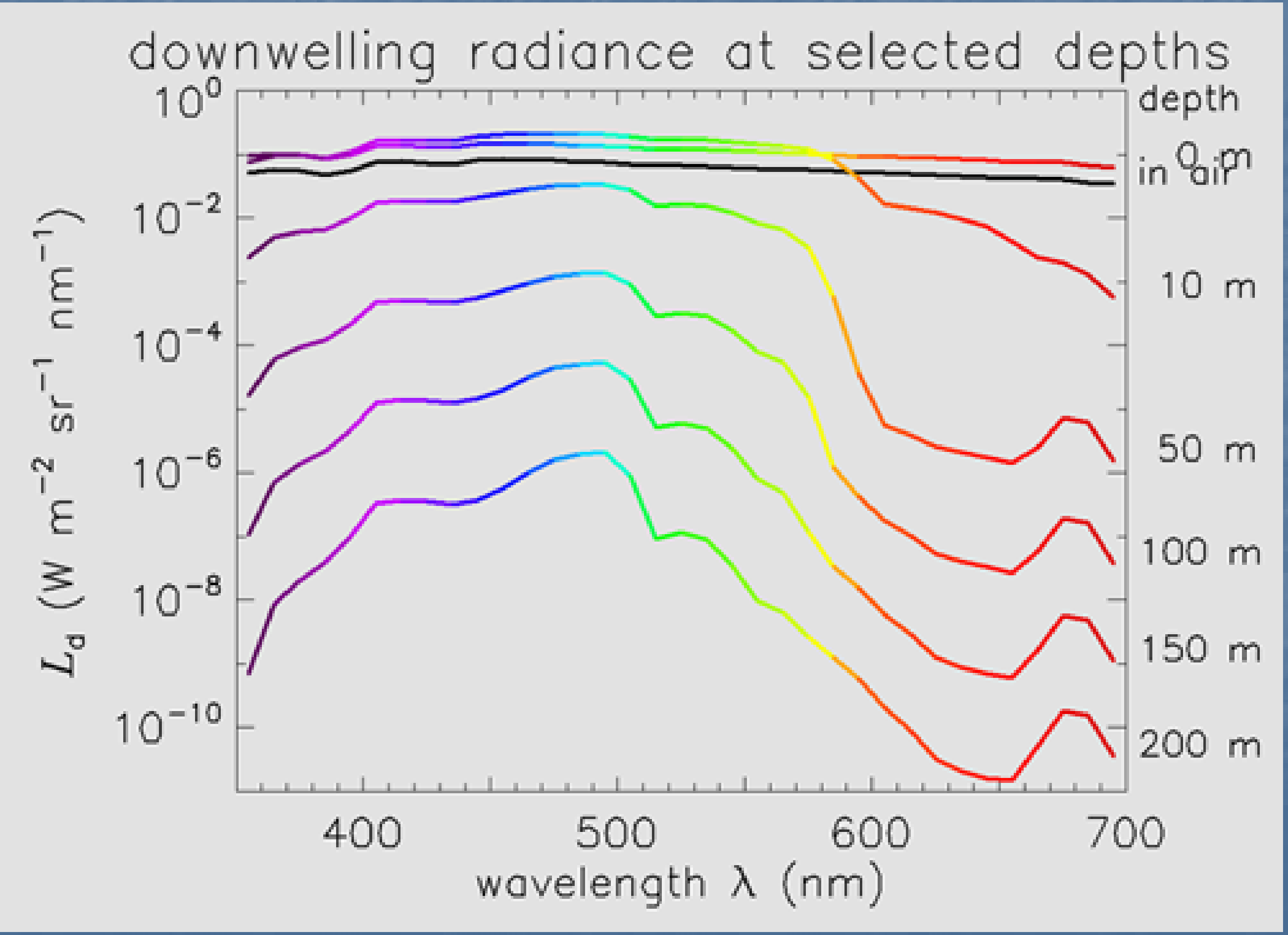
- The different polarizations of the radiance in different directions tend to average out when the radiance is integrated to get irradiance

- We do not have many models or data for the inputs needed to compute polarization in the ocean

Keep in mind, however, that ignoring polarization (e.g., in HydroLight) causes some error (~10% in radiance, ~1% in irradiance) and that use of polarization will become more important in future years, as instruments and models improve.

Radiance is always hard to visualize and plot because of so many variables

Example plot: Radiance $L(z, \theta, \phi, \lambda)$ as a function of z and λ for the zenith-viewing direction (the downwelling radiance L_d: light traveling straight down, detector pointed straight up)



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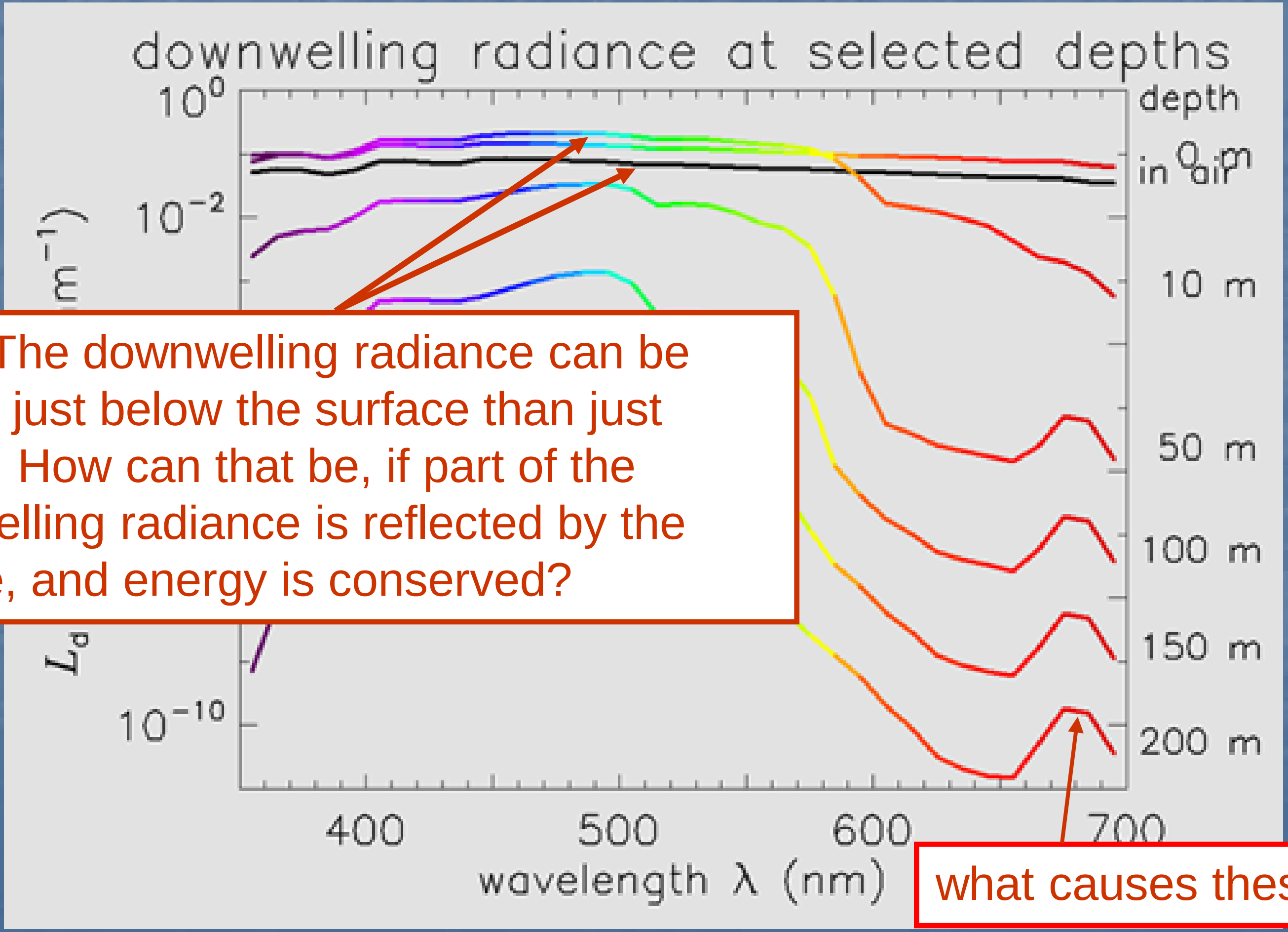
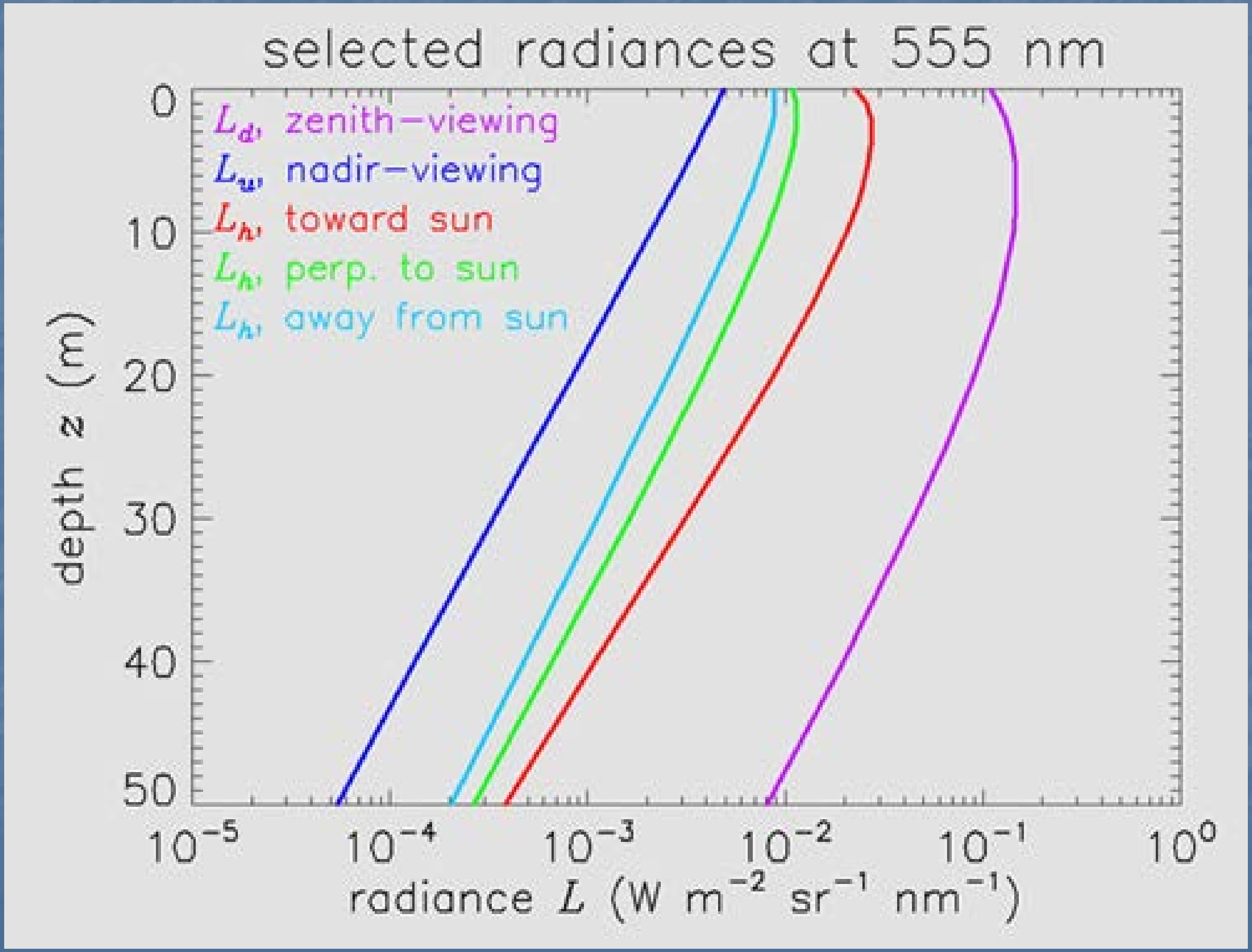


Note: The downwelling radiance can be greater just below the surface than just above. How can that be, if part of the downwelling radiance is reflected by the surface, and energy is conserved?

what causes these bumps?

Example plot: Radiance $L(z, \theta, \phi, \lambda)$ as a function of depth z and selected directions for one wavelength, λ = 555 nm.



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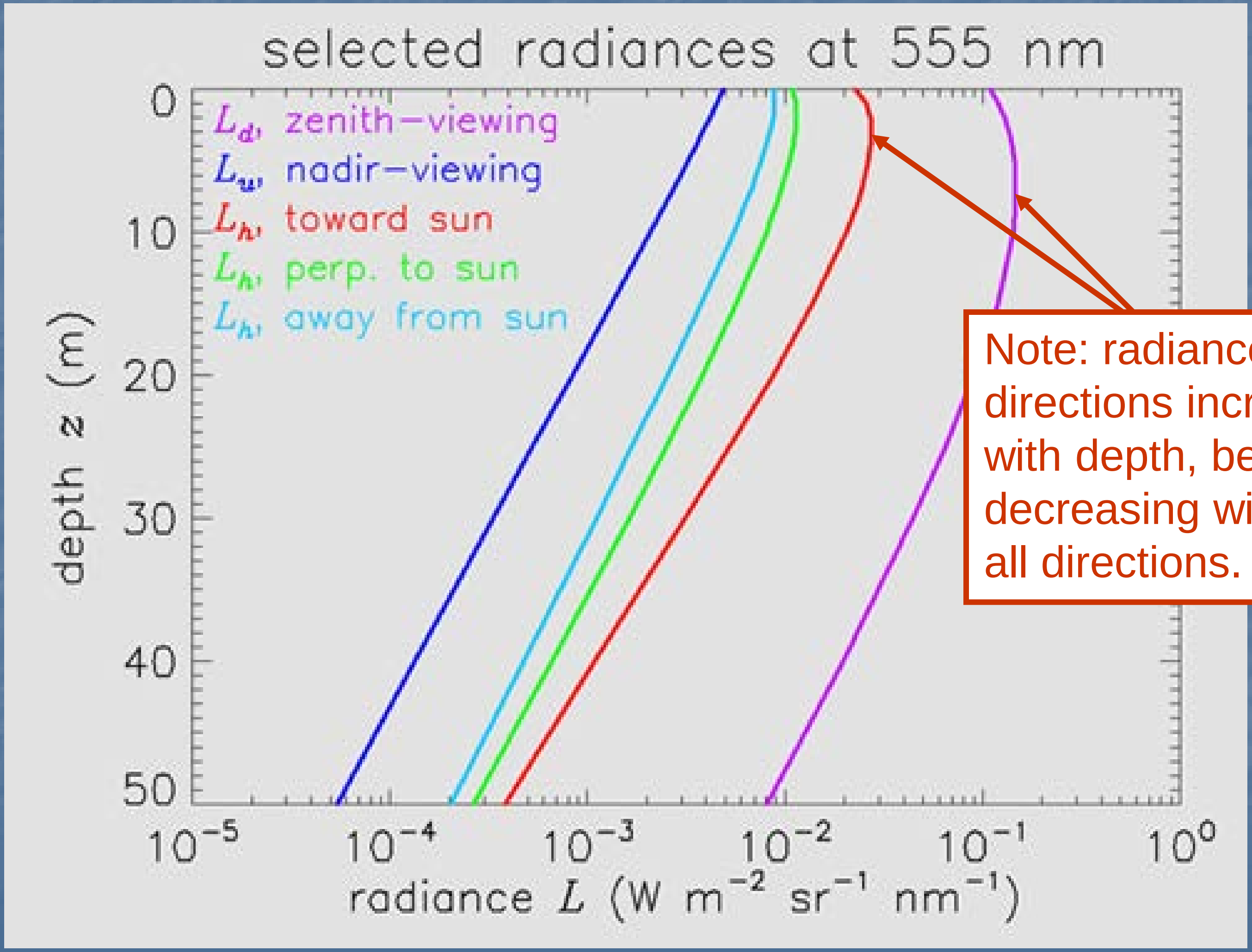
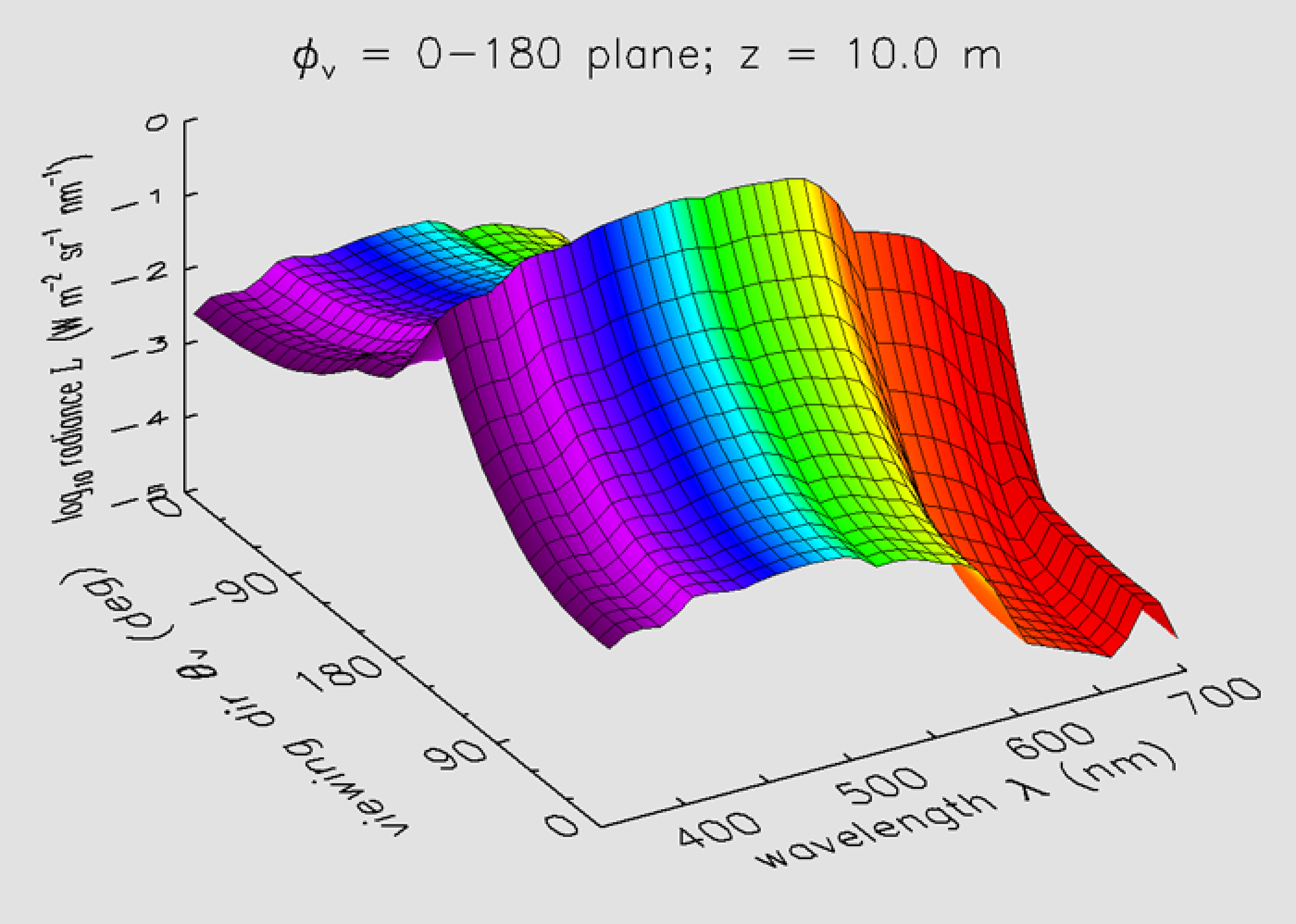


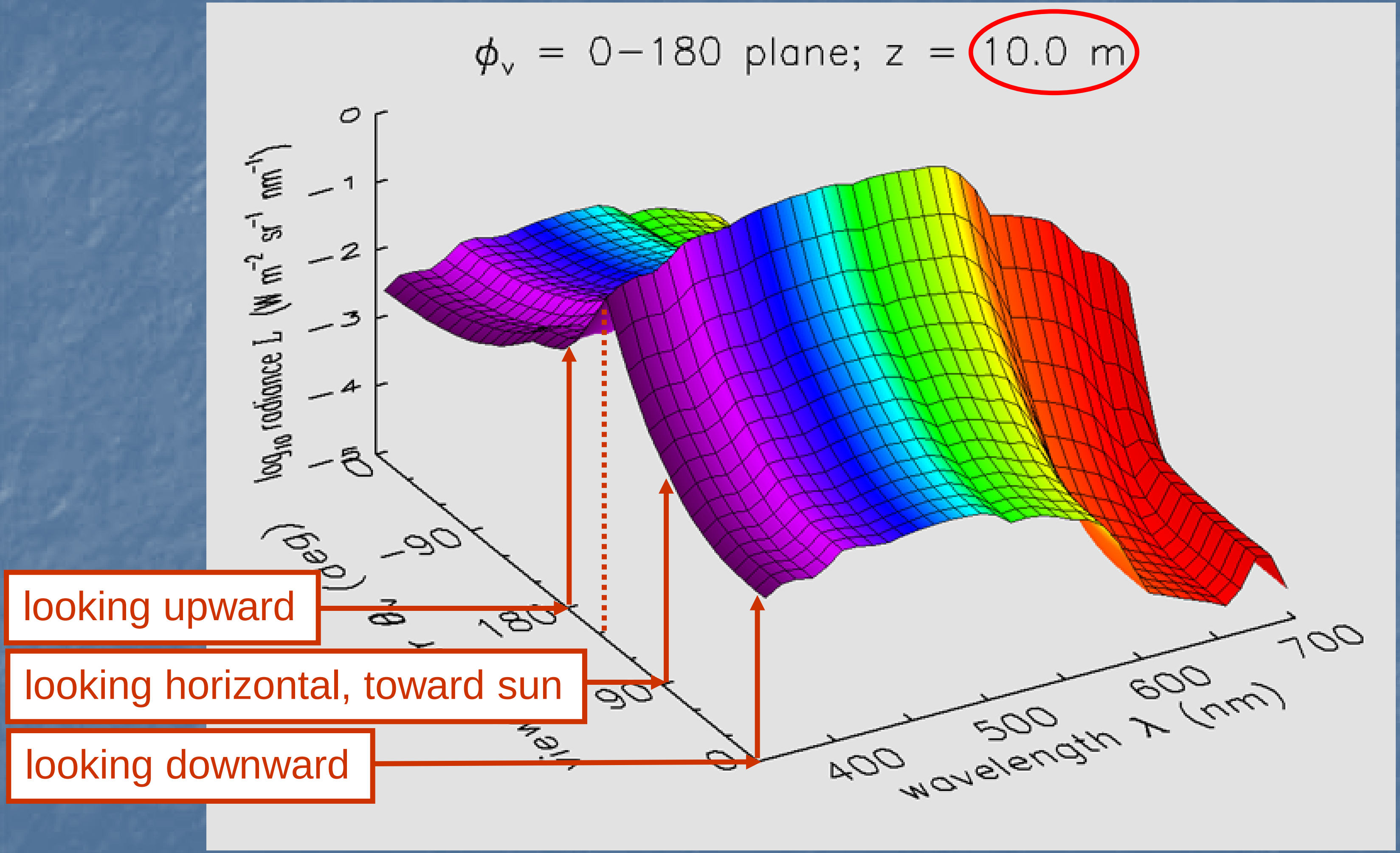
Note: radiance in some directions increases with depth, before decreasing with depth in all directions. Why?

Example plot: Radiance $L(z, \theta, \phi, \lambda)$ as a function of polar angle θ and wavelength λ, for depth z = 10 m and ϕ in the plane of the sun



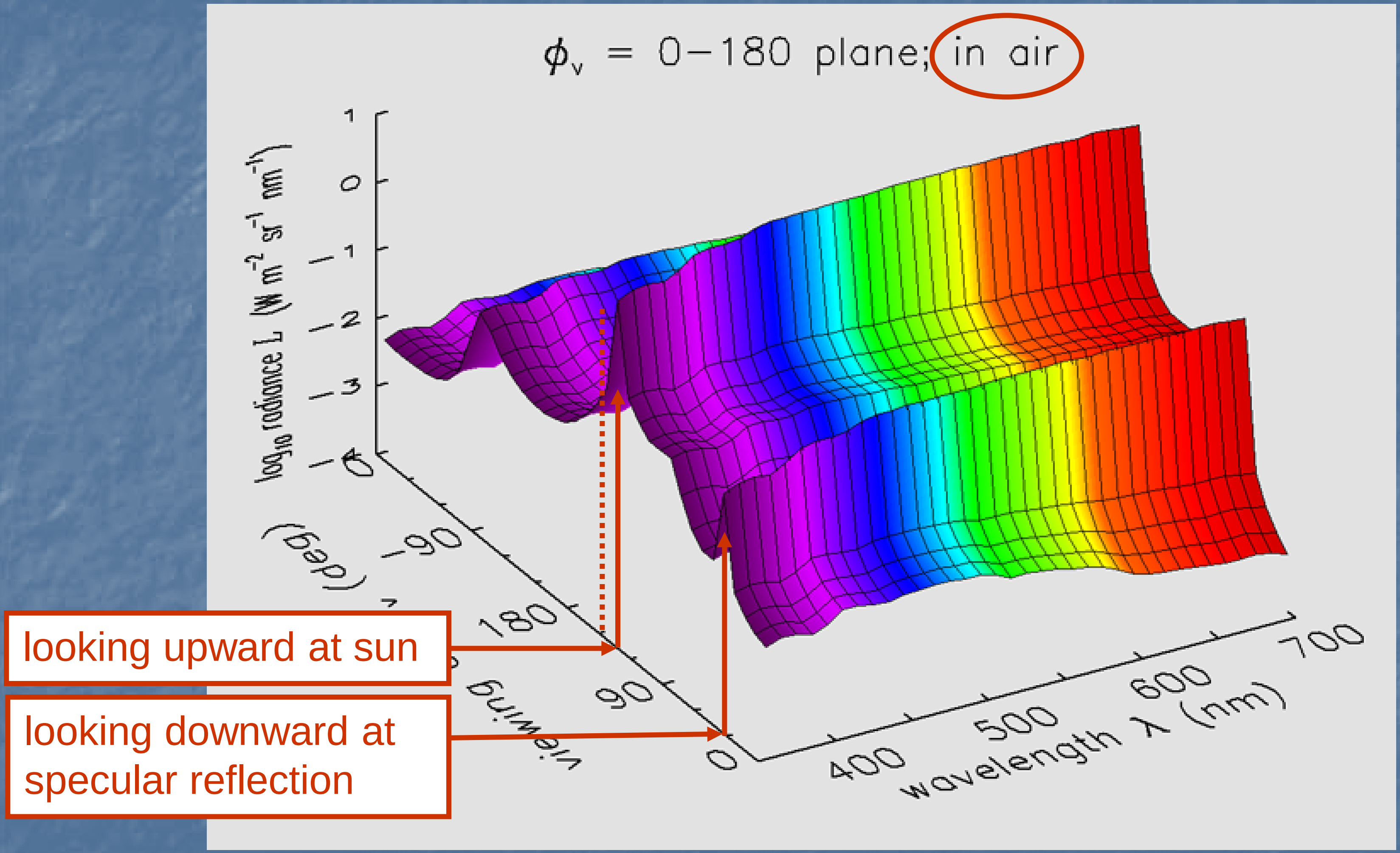
Note: $+z$ is downward, so θ = 0 is light heading straight down, viewed by looking straight up in the θ_v = 180 deg direction.

Example plot: Radiance $L(z, \theta, \phi, \lambda)$ as a function of polar angle θ and wavelength λ, for depth z = 10 m and ϕ in the plane of the sun



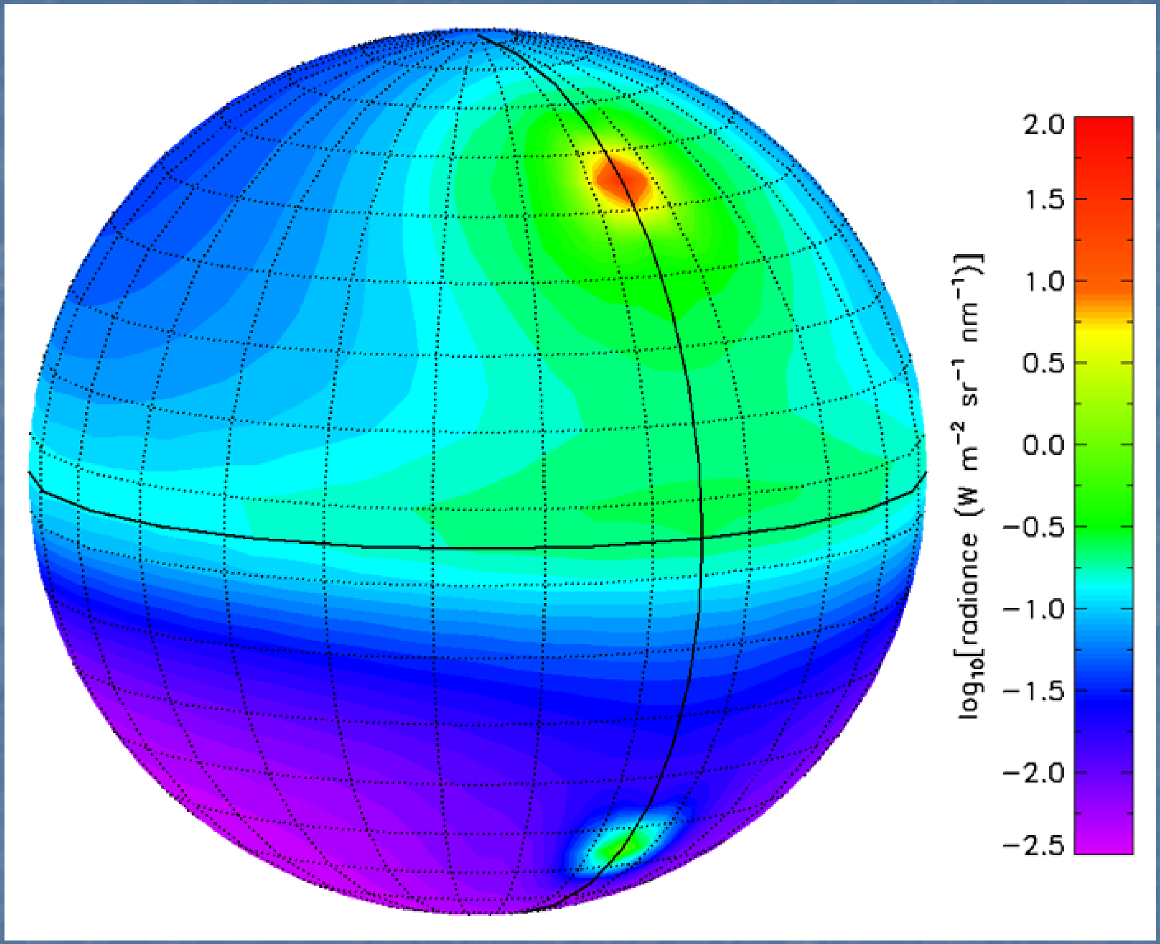
Note: +z is downward, so θ = 0 is light heading straight down, viewed by looking straight up in the θ_v = 180 deg direction.

Example plot: Radiance $L(z, \theta, \phi, \lambda)$ as a function of polar angle θ and wavelength λ, just above the sea surface and ϕ in the plane of the sun



Note: $+z$ is downward, so $\theta = 0$ is light heading straight down, viewed by looking straight up in the θ_v = 180 deg direction.

Example plot: Radiance $L(z, \theta, \phi, \lambda)$ just above the sea surface as a function of θ and ϕ for λ = 555 nm.



See www.oceanopticsbook.info/view/light_and_radiometry/visualizing_radiances for a full discussion of these plots.

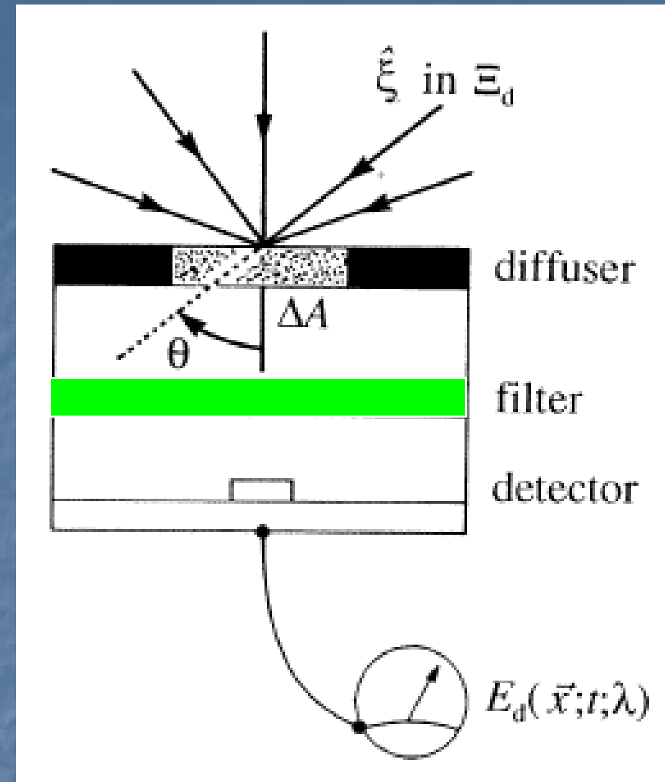
Spectral Plane Irradiance

The most commonly measured radiometric variable

The collector *surface* is equally sensitive to light from any direction.

However, the effective (projected) area of the detector as “seen” by light in direction ¸ is ” A cos(¸).

So must weight the radiance by cos(¸) when computing plane irradiance

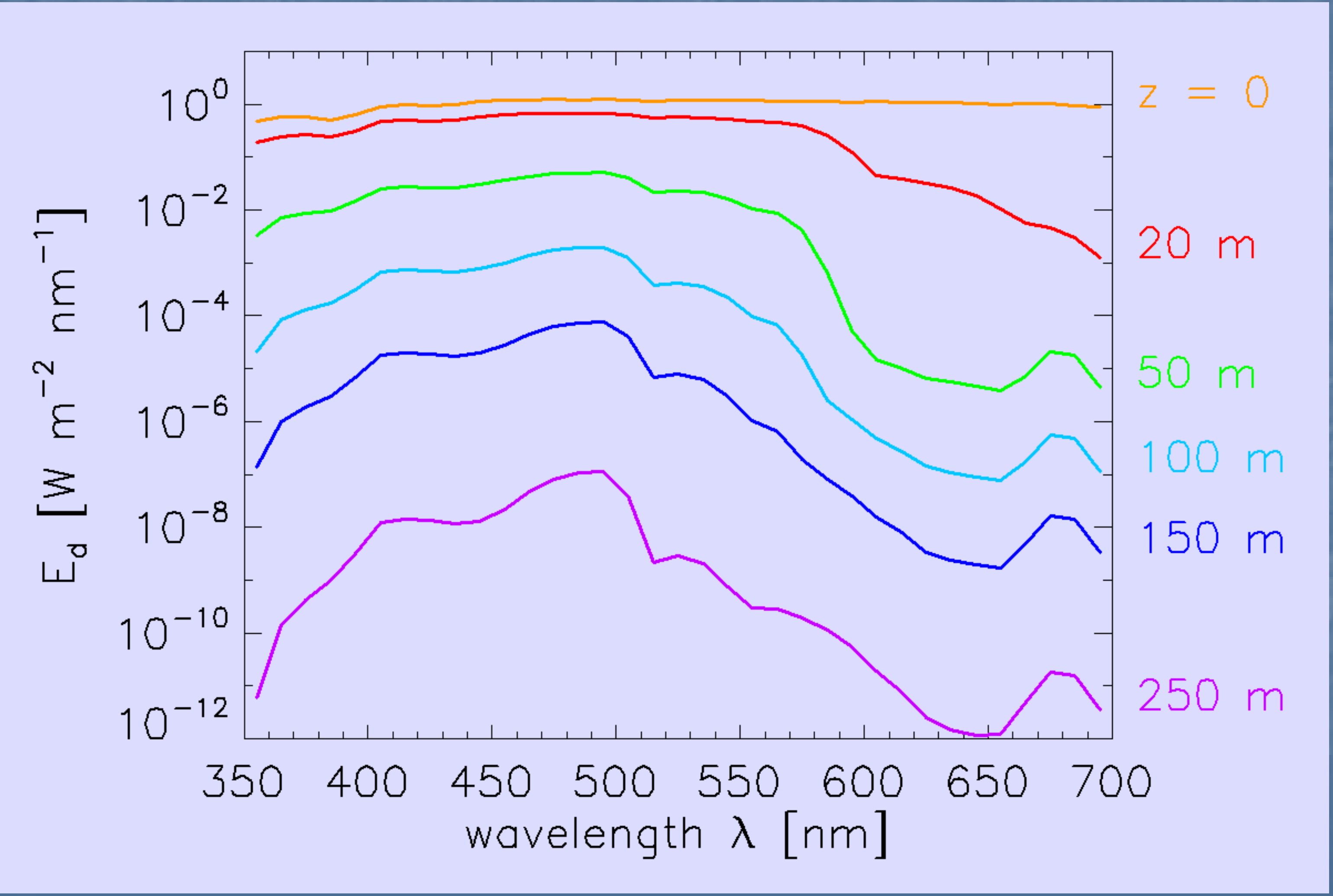


$$E_d(\vec{x}, t, \lambda) \equiv \frac{\Delta Q}{\Delta t \, \Delta A \, \Delta \lambda} \quad (\mathrm{W \ m^{-2} \ nm^{-1}})$$

$$E_d(\vec{x}, t, \lambda) =$$

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L(\vec{x}, t, \theta, \phi, \lambda) |\cos\theta| \, \sin\theta d\theta \, d\phi$$

Example plot: E_d as a function of wavelength for selected depths



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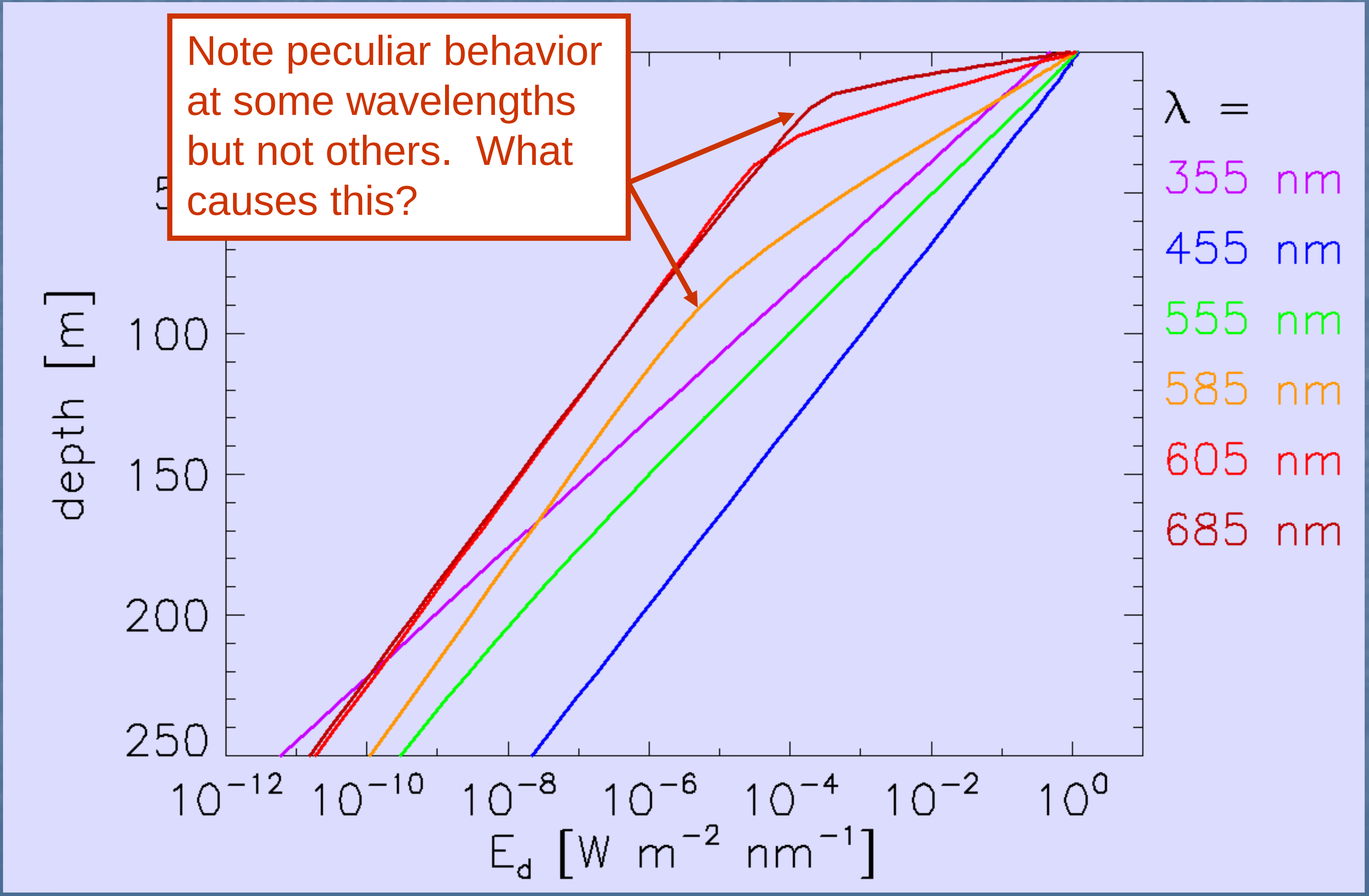
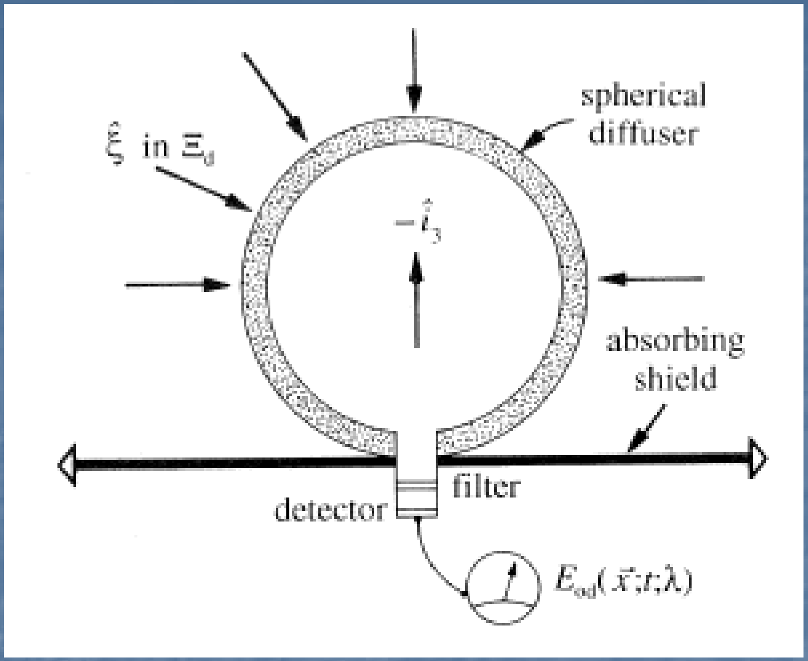


Spectral Scalar Irradiance

The radiometric variable that is most relevant to photosynthesis and water heating because those processes are independent of the direction the light is travelling

The detector has the same effective area for radiance in any downward direction, so no cos(¸) factor when integrating over radiance to get E_o



$$E_{od}(\vec{x},t,\lambda) \equiv \frac{\Delta Q}{\Delta t\,\Delta A\,\Delta\lambda} \quad (\mathrm{W\ m^{-2}\ nm^{-1}})$$

$$E_{od}(\vec{x},t,\lambda) = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} L(\vec{x},t,\theta,\phi,\,\lambda)\,\sin\theta\, d\theta\, d\phi$$

$$E_o(\vec{x},t,\lambda) = E_{od}(\vec{x},t,\lambda) + E_{ou}(\vec{x},t,\lambda)$$

Spectral Vector Irradiance

can be related to absorption by Gershun's law (see *Light and Water* or the web book):

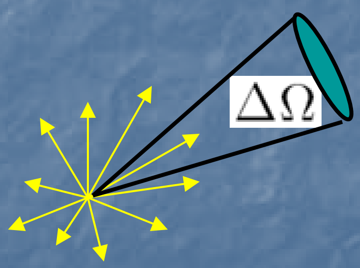
$$
\begin{aligned}
(\vec{E})_z &= \hat{z} \cdot \vec{E} \\
&= \int_{\Xi} L(\vec{x}, t, \hat{\xi}, \lambda) \cos\theta \, d\Omega(\hat{\xi}) \\
&= \int_{\theta=0}^{90} L(\ldots\theta\ldots) \cos\theta \, d\Omega \;+\; \int_{\theta=90}^{180} L(\ldots\theta\ldots) \cos\theta \, d\Omega \\
&= E_d - E_u
\end{aligned}
$$

$a = -(1/E_o)\, d(E_d - E_u)/dz$

$E_z = E_{net} = E_d - E_u$ is the *net downward irradiance*

Spectral Intensity

useful for describing *point* light sources



$$I(\vec{x}, t, \hat{\xi}, \lambda) = \frac{\Delta Q}{\Delta t \, \Delta\Omega \, \Delta\lambda}$$

$$(\mathrm{W\ sr^{-1}\ nm^{-1}})$$

Photosynthetically Available Radiation (PAR)

Historically used in simple models for phytoplankton growth

More sophisticated models today use the spectral scalar quantum irradiance because different phytoplankton pigments absorb light differently at different wavelengths.

RADIOMETRY uses ENERGY units

PHOTOSYNTHESIS depends on the NUMBER on photons absorbed. The convenient measure of how many photons are available for photosynthesis is

$$PAR \equiv \int_{400\,\mathrm{nm}}^{700\,\mathrm{nm}} E_o(\lambda)\,\frac{\lambda}{hc}\,d\lambda$$
$$(\mathrm{photons}\ \mathrm{s}^{-1}\ \mathrm{m}^{-2})$$

PAR is often expressed as Einsteins s^{-1} m^{-2}

1 Einstein = 1 mole of photons
= 6.023 x 10^{23} photons

Warnings on Terminology

In atmospheric optics, radiance is often called “specific intensity” and irradiance is called “flux”. Some people call irradiance “flux” and some call irradiance “flux density”. Other fields (medical optics, astrophysics, etc.) have their own terminiolgy and notation (e.g., in medicine “fluence” is energy/area, “fluence rate” is irradiance).

spectral vs band-integrated radiance and irradiance:

Spectral downwelling plane irradiance $E_d(\lambda)$ is per unit wavelength interval, with units of W m^{-2} nm^{-1}

Band-integrated downwelling plane irradiance is the spectral irradiance integrated over some finite wavelength band, with units of W m^{-2}, e.g.,

$$E_d = \int_{410}^{420} E_d(\lambda)\, d\lambda$$

It is often hard to figure out exactly what is being measured or discussed in a paper. Units and magnitudes matter! I reject papers that are not clear or have inconsistent or wrong units.

Sea Kayaking in Panama, Feb 2012

photo by Curtis Mobley