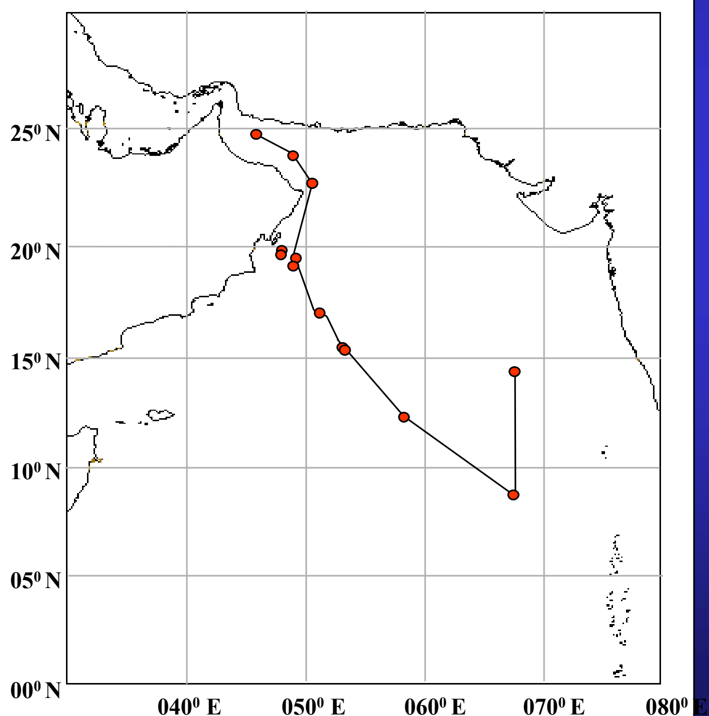
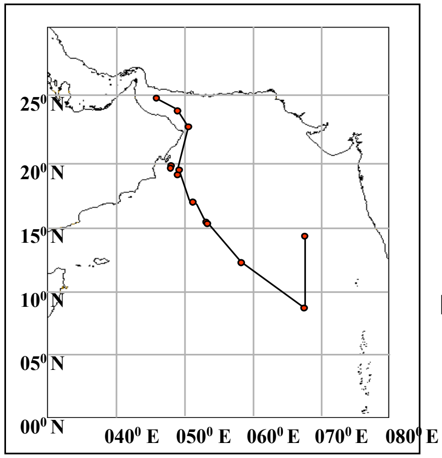
LECTURE 2

MODELLING PP ON LOCAL SCALES

CRUISE TRACK AND STATION POSITIONS
R.R.S. DISCOVERY CRUISE
D212



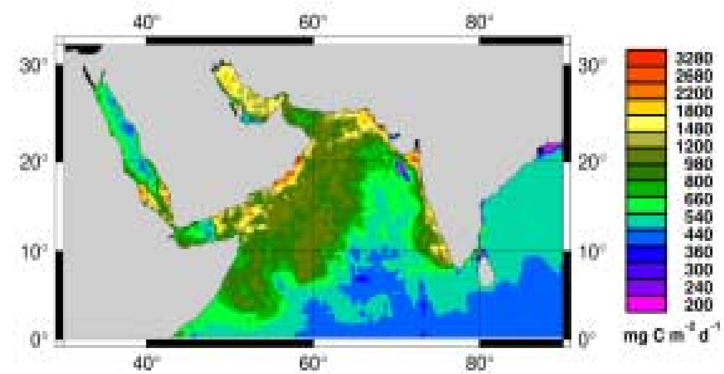
CRUISE TRACK AND
STATION POSITIONS
R.R.S. DISCOVERY CRUISE
D212



LECTURE 3

MODELLING PP AT OCEAN BASIN SCALES

DAILY INTEGRATED PRIMARY PRODUCTION



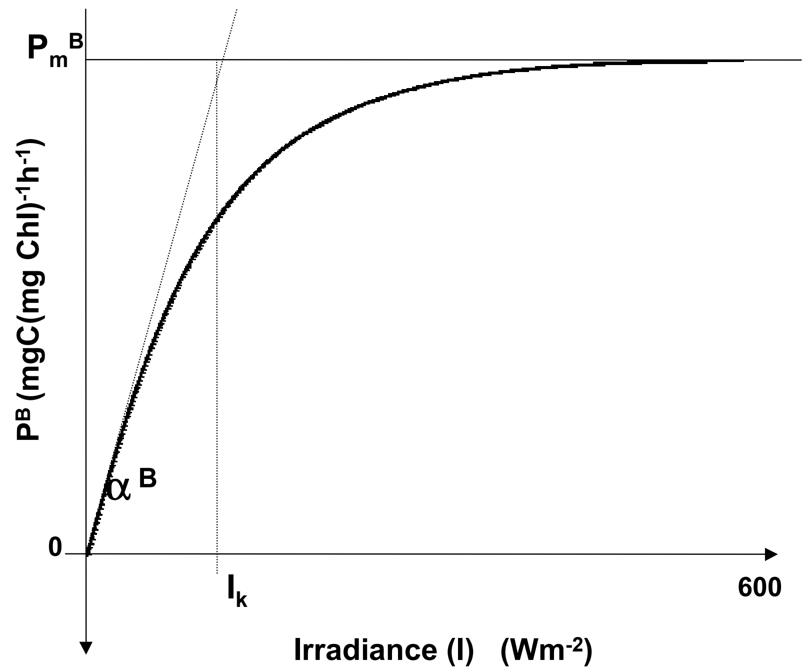
LECTURE 2

MODELLING PP ON LOCAL SCALES

"WHAT'S THE PROBLEM?"

HOW TO MODEL DAILY INTEGRATED PRIMARY PRODUCTION OVER THE OCEAN'S WATER COLUMN AT A PARTICULAR LOCATION ON A PARTICULAR DAY

THE PHOTOSYNTHESIS-LIGHT CURVE



P^B_m = Maximum assimilation number (normalised to B)

α^B = Initial slope

The P-I curve can be represented by an equation of the form:

$$P^B = p(I; \alpha^B, P^B_m)$$

BASICS:

MATHEMATICAL REPRESENTATION OF THE PROBLEM

Normalised production P^B is defined as:

$$P^B=P/B \qquad (1)$$

Absolute primary production can be recovered by :

$$P=B \times P^B \qquad (2)$$

Incorporating depth dependency:

$$P(z)=B(z)\,P^B(z) \qquad (3)$$

The P/I curve:

$$P^B(z) = p(I(z), \text{parameters})$$

Incorporating the P/I curve:

$$P(z)=B(z)p(I(z),\text{parameters}) \qquad (4)$$

Introducing daylength D (hours):

I(*z*) is a time-dependent variable

$$P_{Z,T}= \int_0^D \int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt. \qquad (5)$$

Equation (5) is the statement of our problem.

PROBLEM:

$$P_{Z,T}= \int_0^D \int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt. \qquad (5)$$

HOW WE MODEL DAILY INTEGRATED PRIMARY PRODUCTION OVER THE OCEAN'S WATER COLUMN AT A PARTICULAR LOCATION ON A PARTICULAR DAY

ESTABLISHING A "LOCAL ALGORITHM" - CONSIDER TWO POSSIBLE APPROACHES (T. Platt and S. Sathyendranath and co-workers):

METHOD I - RELATIVELY SIMPLE -

- I(z,t) IS NOT WAVELENGTH DEPENDENT
- B(z) IS CONSTANT ie BIOMASS PROFILE IS UNIFORM

REQUIRES *ANALYTIC MODEL*

METHOD II - MORE COMPLEX

REQUIRES *NUMERICAL MODEL* = MORE COMPUTING TIME

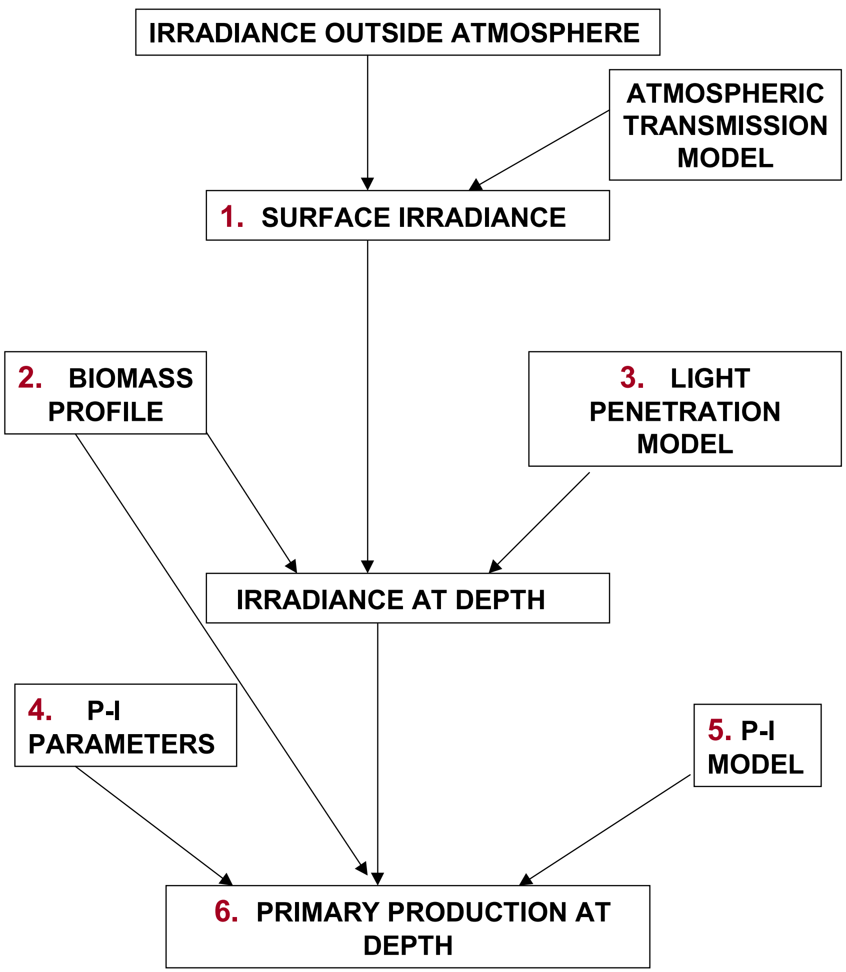
- I(z,t) IS WAVELENGTH DEPENDENT i.e. SPECTRALLY RESOLVED
- B(z) IS NOT CONSTANT ie BIOMASS PROFILE IS NON-UNIFORM

BOTH *METHODS* ARE BASED ON EQ. 5

$P_{Z,T}=\int_0^D\int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt.$ (5)

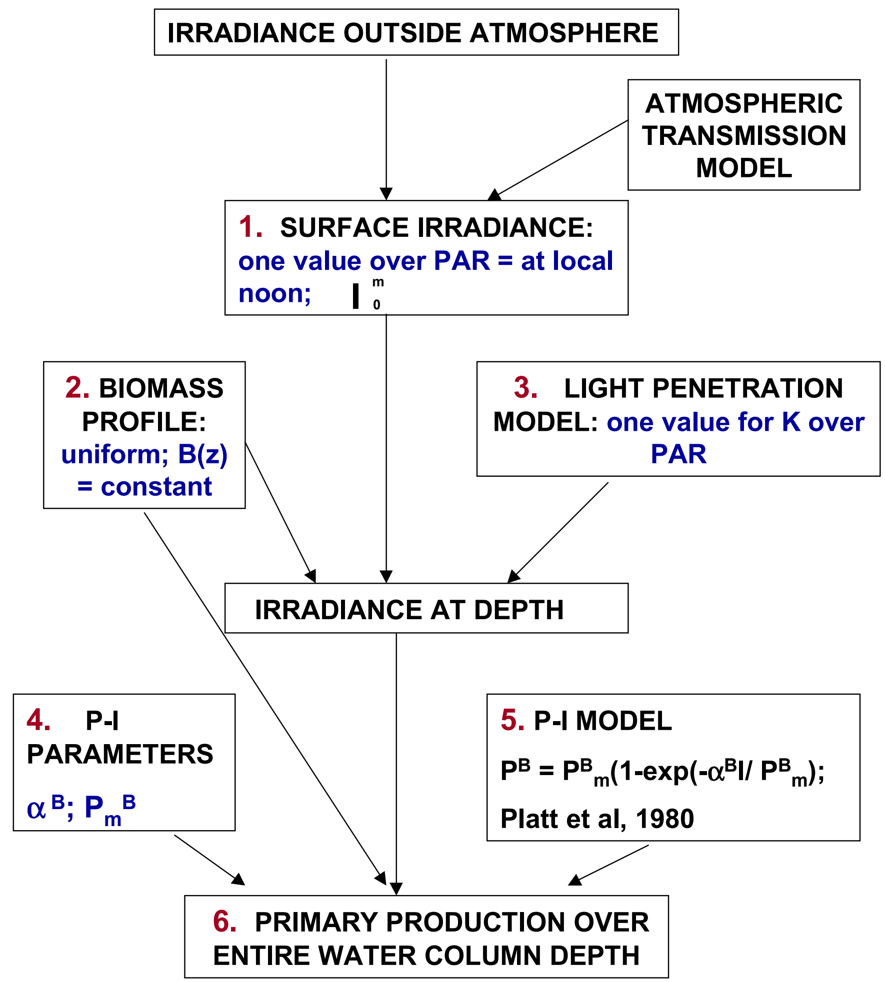
COMMON STEPS INVOLVED IN *METHODS I & II*:

Computation of PP at depth



PROBLEM: $P_{Z,T}=\int_0^D\int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt.$ (5)

METHOD I: wavelength independent and uniform biomass: ANALYTIC MODEL



METHOD I: ***wavelength independent and uniform biomass: ANALYTIC MODEL***

SOFTWARE: INPUTS REQUIRED FROM USER:

- **latitude, longitude, day number - allows computation of I_0^m for step 1, the computation of sea-surface irradiance**

- **Biomass B for step 2: constant value because assuming uniform profile.**

- **The specific attenuation coefficient, K for step 3: constant value averaged over PAR.**

- **P-I parameters: α^B, P^B for steps 4 and 5: - assuming broad-band values for both and independent of depth.**

- **Lower bound (e.g. Z=0m), upper bound (Z=100)**

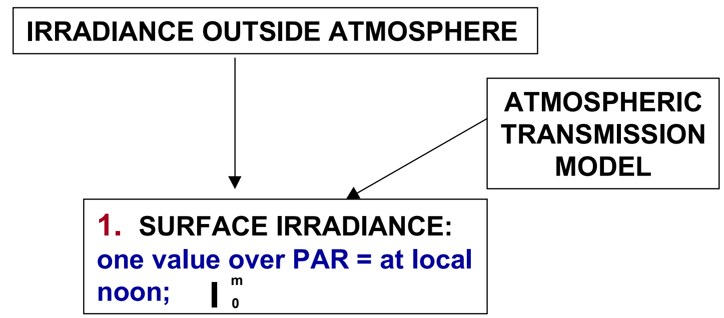
SOFTWARE: OUTPUTS

- **D (daylength hours)**
- **I_*^m (normalised midday irradiance) (dimensionless)**
- **DAILY PRIMARY PRODUCTION OVER THE LAYER (mg C m^{-2})**

PROBLEM: $P_{Z,T}=\int_0^D\int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt$ **(5)**

METHOD I: ***wavelength independent and uniform biomass: ANALYTIC MODEL***

STEP 1:



Involves:

- **Computation of sunrise (a function of latitude)**
- **Computation of daylength, D (hours) - a function of latitude and solar declination (Iqbal, 1983)**
- **Computation of local noon time from sunrise + D/2**
- **Computation of zenith angle of sun at local noon (Paltridge and Platt, 1976)**
- **Input of Zenith angle into an atmospheric transmission model (Bird 1984) for computation of total irradiance (direct + diffuse) over PAR hitting sea-surface: I_0^m**

I_0^m is normalised to I_*^m

(I_k is calculated from PI parameters)

PROBLEM: $P_{Z,T} = \int_0^D \int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt.$ (5)

METHOD I: wavelength independent and uniform biomass:
ANALYTIC MODEL

STEP 2:

2. BIOMASS PROFILE: uniform; B(z) = constant

Calculated as average biomass value either from uniform or non-uniform profile

STEP 3:

3. LIGHT PENETRATION MODEL: one value for K over PAR

Calculated from light profile:

$I(z) = I_0 e^{-Kz}$

STEP 4:

4. P-I PARAMETERS α^B; P_m^B

Broad-band values from shipboard measurements

STEP 5:

5. P-I MODEL $P^B = P^B_m(1-\exp(-\alpha^B I/ P^B_m)$; Platt et al, 1980

METHOD I: wavelength independent and uniform biomass:
ANALYTIC MODEL

DERIVATION OF THE SOLUTION USED IN STEP 6

6. PRIMARY PRODUCTION OVER ENTIRE WATER COLUMN DEPTH

PROBLEM:

$$P_{Z,T} = \int_0^D \int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\ dt. \quad (5)$$

To **find the analytic solution** substitute:

p(I(*z*,t), parameters) by the chosen equation for the P/I curve:

$$P/B = P^B = P^B_m(1-\exp(-\alpha^B I(z,t)/ P^B_m)$$

And **I(*z*)** by:

$$I(z) = I_0 e^{-Kz}$$

And describing the sinusoidal variation of I with time, substitute I_0 by:

$$I_0(t) = I_0^m \sin(\pi\ t/D)$$

RESULT: a complex expression!!

$$P_{Z,T} = B\ P^B_m \int_0^D \int_0^\infty (1-\exp[-(\alpha^B\ I_0^m \sin(\pi\ t/D)e^{-Kz})/ P^B_m)\ dz\ dt.$$

DIMENSIONAL ANALYSIS

It is based on the idea that when a relationship is found between the property of interest and the variables upon which it depends, that relation must be **dimensionally consistent**.

Back to Eq (5):

PROBLEM: $P_{Z,T} = \int_0^D \int_0^\infty B(z)\ p(I(z,t), \text{parameters})\ dz\, dt.$

UNITS $P_{Z,T}$ = mg C m^{-2} d^{-1}

UPON WHICH VARIABLES DOES $P_{Z,T}$ DEPEND?

$I_0^m \quad K \quad B \quad \alpha^B \quad P^B_m \quad D$

$P_{Z,T} \sim B^q \quad (\alpha^B)^u \quad (P^B_m)^v \quad D^w \quad (I_0^m)^x \quad K^y$

Where q, u, v, w, x, y are exponents yet to be determined

DIMENSIONS

Let: [B] = mass of biomass ie chlorophyll (e.g. mg)
[C] = mass of carbon (e.g. mg)
[L] = length (e.g. m)
[T] = time (e.g. h)
[F] = flux of photons (e.g. Wm^{-2})

$P_{Z,T} \sim B^q \ (\alpha^B)^u \ (P^B_m)^v \ D^w \ (I_0^m)^x \ K^y$

UNITS LHS: $P_{Z,T}$ = mg C m^{-2}

REPLACING EVERY TERM BY ITS DIMENSIONS:

$$[CL^{-2}] = [BL^{-3}]^q\, [CB^{-1}T^{-1}F^{-1}]^u\, [CB^{-1}T^{-1}]^v\, [T]^w\, [F]^x\, [L^{-1}]^y$$

$$\therefore\ [CL^{-2}] = [B]^{q-u-v}\, [C]^{u+v}\, [T]^{w-u-v}\, [F]^{x-u}\, [L]^{-y-3q}$$

DIMENSIONS RHS MUST EQUAL DIMENSIONS LHS:

$q-u-v = 0$	5 simultaneous equations	$q = 1$
$u+v = 1$	→	$y = -1$
$w-u-v = 0$		$w = 1$
$x-u = 0$	x remains unknown	$u = x$
$-y-3q = -2$		$v = 1-x$

DIMENSIONAL ANALYSIS

SUBSTITUTING:

$q= 1,\ y=-1,\ w=1,\ u=x,\ v=1-x$

into

$$P_{Z,T} \sim B^q\ (\alpha^B)^u\ (P^B{}_m)^v\ D^w\ (I_0{}^m)^x\ K^y$$

we get:

$$P_{Z,T} \sim \{(B\ P^B{}_m\ D)\ /\ K\}f(\ I_0{}^m\alpha^B/P^B{}_m)$$

From the P-I curve we know:

$$1/\ I_k\ = \alpha^B/\ P^B{}_m$$

$$\therefore\ P_{Z,T} \sim \{(B\ P^B{}_m\ D)\ /\ K\}f(\ I_0{}^m/\ I_k)$$

$I_0{}^m/\ I_k$ is the normalised midday irradiance; $I_*{}^m$

$$\therefore\ P_{Z,T} \sim \{(B\ P^B{}_m\ D)\ /\ K\}f(I_*{}^m)$$

This is the general form that the analytic solution must take.

It consists of a factor; $\{(B\ P^B{}_m\ D)\ /\ K\}$ multiplied by a dimensionless function of the scaled irradiance $f(I_*{}^m)$

DIMENSIONAL ANALYSIS

INTEGRATING ANALYTIC SOLUTION:

$$P_{Z,T} = B\ P^B{}_m \int_0^D \int_0^\infty (1-\exp[-(\alpha^B\ I_0{}^m\ \sin(\pi\ t/D)e^{-KZ})/\ P^B{}_m)\ dz\ dt.$$

SOLUTION:

$$P_{Z,T} = \{(B\ P^B{}_m\ D)\ /\ K\}\ x$$

$$\left[\sum_{n=1}^{\infty}\frac{2(I_*^m)^{2n-1}}{\pi(2n-1)(2n-1)!}\frac{(2n-2)!!}{(2n-1)!!}-\sum_{n=1}^{\infty}\frac{(I_*^m)^{2n}}{2n(2n)!}\frac{(2n-1)!!}{(2n)!!}\right]$$

The analytic solution therefore has the form anticipated by the dimensionless analysis:

$$P_{Z,T} \sim \{(B\ P^B{}_m\ D)\ /\ K\}f(I_*{}^m)$$

It consists of a factor $=\{(B\ P^B{}_m\ D)\ /\ K\}$ multiplied by a dimensionless function

Values for $f(I_*{}^m)$ can therefore be calculated for a range of $I_*{}^m$ values from the above expression and these are tabulated.

For a known value of $I_*{}^m$ we can therefore calculate $P_{Z,T}$

DIMENSIONAL ANALYSIS

ALTERNATIVELY can use:

$$P_{z,T} = A \sum_{X=1}^{5} \Omega_x (I_*^m)^X$$

where $A = \{(B\, P^B_m\, D) / K\}$

This approximates to the same as the Analytic solution (Platt et al, 1990) but is more straightforward!

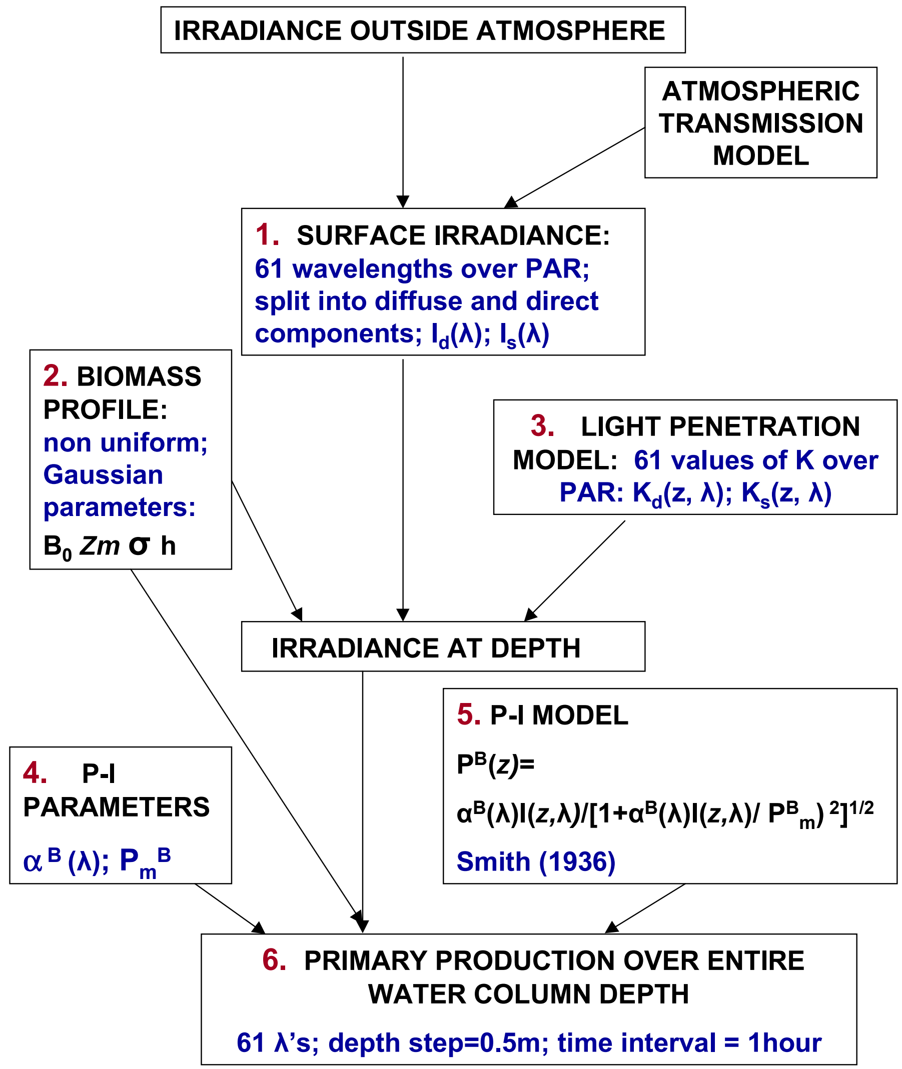
EXPANSION OF THE 5th ORDER POLYNOMIAL GIVES:

$$P_{z,T} = ((B\, P^B_m\, D) / K) \times \{\Omega_5 (I_*^m)^5 + \Omega_4 (I_*^m)^4 + \Omega_3 (I_*^m)^3 + \Omega_2 (I_*^m)^2 + \Omega_1 (I_*^m)^1\}$$

Where: Ω_1 **to** Ω_5 **are coefficients of the polynomial expression and are tabulated for a known value of** I_*^m **(Platt and Sathyendranath 1993)**

This equation is used in the software STEP 6 in *METHOD I*

METHOD II: spectrally resolved and non uniform biomass profile: NUMERICAL MODEL



METHOD II: spectrally resolved and non-uniform biomass profile: NUMERICAL MODEL

SOFTWARE: INPUTS REQUIRED FROM USER:

- For step 1- the computation of sea-surface irradiance: -
  latitude; longitude; day number;
  cloud cover (%)
- For step 2-the computation of the biomass profile:-
  Gaussian parameters:
  peak depth; Zm
  baseline biomass; B_0
  std deviation around the peak; σ
  total biomass within the peak; h
- For step 3- the computation of the diffuse and direct attenuation coefficients:-

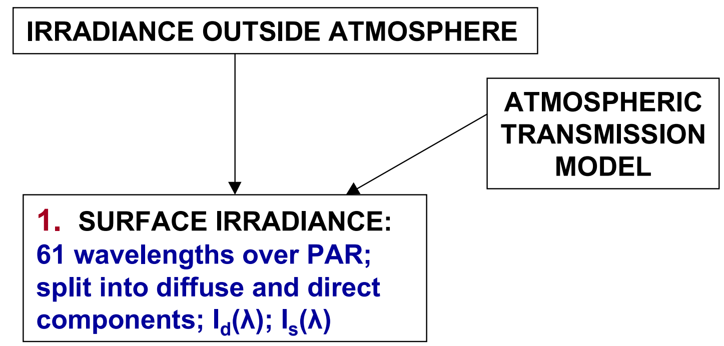
  Ay(440)/Ac(440) ie the proportion of the absorption of light by yellow substances (CDOM).
- For steps 4 and 5- the P-I parameters:
  - $\alpha^B(\lambda)$, P^B_m
- Also input the bottom depth

SOFTWARE: OUTPUTS

- Daily primary production ; Photic zone depth (Zp).

METHOD II: spectrally resolved and non uniform biomass profile: NUMERICAL MODEL

STEP 1:



Involves:

- Computation of sunrise (a function of latitude)
- Computation of daylength, D (hours) - a function of latitude and solar declination (Iqbal, 1983)
- Computation of local noon time from sunrise + D/2
- Computation of zenith angle of sun every hour from sunrise to local noon (Paltridge and Platt, 1976)
- Input of Zenith angle into an atmospheric transmission model (Bird 1984) for computation of sea-surface irradiance at 61 λ's over PAR, split into direct & diffuse components, for every hour (assumes symmetry around noon).

METHOD II: spectrally resolved and non-uniform biomass profile: NUMERICAL MODEL

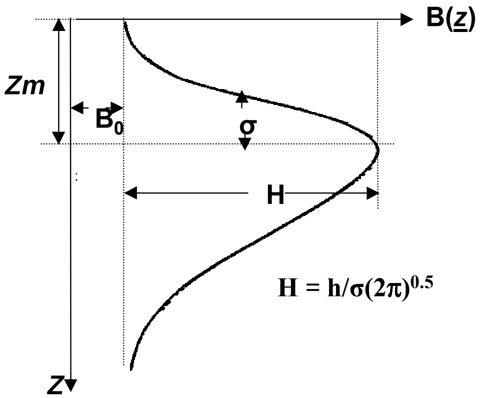
STEP 2:

2. BIOMASS PROFILE:

non-uniform; described by shifted Gaussian:

$B(z) =$

$B_0 + \{(h/(\sigma 2\pi^{1/2}))\ \exp[-(z-z_m)^2/2\sigma^2]\}$



B_0 = the background pigment (mg m^{-3})

***Zm* = the depth of the chlorophyll peak (m)**

σ = the standard deviation around the peak value (m)

h = the total pigment within the peak (mg m^{-2})

H= the height of the peak above the background (m)

METHOD II: spectrally resolved and non-uniform biomass profile: NUMERICAL MODEL

STEP 3:

3. LIGHT PENETRATION MODEL: 61 values of K over PAR: $K_d(\lambda,z)$; $K_s(\lambda,z)$

$$K_d(z,\lambda) = (a(z,\lambda) + b_b(z,\lambda))/\cos\theta_d$$
$$K_s(z,\lambda) = (a(z,\lambda) + b_b(z,\lambda))/\cos\theta_s$$

$K_d(z,\lambda)$ = K for direct irradiance at depth z and wavelength λ

$K_s(z,\lambda)$ = K for diffuse irradiance at depth z and wavelength λ

$a(z,\lambda)$ = *total* absorption coefficient at z and λ

$b_b(z,\lambda)$ = *total* backscattering coefficient z and λ

$\cos\theta_d$ = the cosine of sun zenith angle in water

$\cos\theta_s$ = the mean cosine of the zenith angles of the diffuse light after refraction at the sea surface

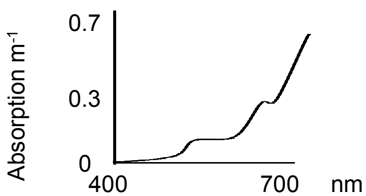
ASSUMING CASE 1 CONDITIONS:

Total absorption coefficient; $a(z,\lambda)$

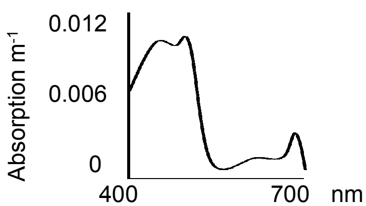
$$a(z,\lambda) = a_w(\lambda) + a_c(\lambda) + a_y(\lambda)$$

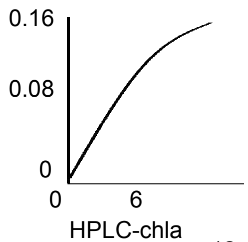
Where:

$a_w(\lambda)$ is the absorption coefficient of pure water (m^{-1}):
(Pope and Fry, 1997. Applied Optics Vol 36 No. 33)



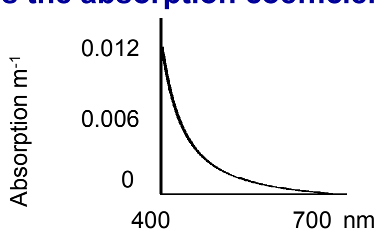
$a_c(\lambda)$ is the absorption coefficient of phytoplankton (m^{-1})





This is estimated using ship board absorption data:

$a_y(\lambda)$ is the absorption coefficient of the CDOM (m^{-1})



ASSUMING CASE 1 CONDITIONS:

Total backscattering coefficient; $b_b(z,\lambda)$

$$b_b(z,\lambda) = b_w(z,\lambda) + b_{bc}(z,\lambda)$$

$b_w(z,\lambda)$ = backscattering coefficient of pure seawater (m^{-1})

Known (Morel, 1974)

$b_{bc}(z,\lambda)$ = backscattering coefficient for phytoplankton (m^{-1})

Computed as a function of the biomass concentration and the wavelength of interest (Loisel and Morel, 1998)

Having computed total absorption and backscattering coefficients we can now calculate the direct and diffuse attenuation coefficients at each depth and for each of the 61 wavelengths

$$K_d(z,\lambda) = (a(z,\lambda) + b_b(z,\lambda))/\cos\theta_d$$
$$K_s(z,\lambda) = (a(z,\lambda) + b_b(z,\lambda))/\cos\theta_s$$

The diffuse and direct light available per depth and wavelength is then computed from:

$$I_d(z,\lambda) = I(0)e^{-Kd(z,\lambda)\,z}$$

$$I_s(z,\lambda) = I(0)e^{-Ks(z,\lambda)\,z}$$

METHOD II: spectrally resolved and non uniform biomass profile: NUMERICAL MODEL

STEP 4:

4. P-I PARAMETERS

$\alpha^B(\lambda)$; P_m^B

BOTH α^B AND P_m^B ARE ASSUMED INDEPENDENT OF DEPTH

P_m^B IS ASSUMED SPECTRALLY NEUTRAL- MODEL REQUIRES ONE BROAD-BAND VALUE FOR P_m^B INPUT BY THE USER.

ONE BROAD-BAND VALUE OF α^B IS REQUIRED AS INPUT- THIS IS USED TO SCALE THE SPECTRALLY RESOLVED ABSORPTION PROPERTIES OF THE PHYTOPLANKTON TO GIVE

$\alpha^B(\lambda)$ - SPECTRALLY RESOLVED α^B

Kyewalyanga MN, Platt T, Sathyendranath S (1997) MEPS: Vol 146: 207-223

METHOD II: spectrally resolved and non uniform biomass profile: NUMERICAL MODEL

STEP 5:

5. P-I MODEL

$P^B(z)=$
$\alpha^B(\lambda)I(z,\lambda)/[1+\alpha^B(\lambda)I(z,\lambda)/P^B_m)^2]^{1/2}$

Smith (1936)

STEP 6:

$P^B(z)= \alpha^B(\lambda)I(z,\lambda)/[1+\alpha^B(\lambda)I(z,\lambda)/P^B_m)^2]^{1/2}$ Smith (1936)

If we let;

$$\Pi^B_\lambda(z) = \alpha^B(\lambda)\, I(z,\lambda)$$

and substitute from step 3:

$I_d(z,\lambda) = I(0)e^{-Kd(z,\lambda)\,Z}$

$I_s(z,\lambda) = I(0)e^{-Ks(z,\lambda)\,Z}$

then:

$$P^B(z)= \Pi^B_\lambda(z) / [1+ \Pi^B_\lambda(z) / P^B_m)^2]^{1/2}$$

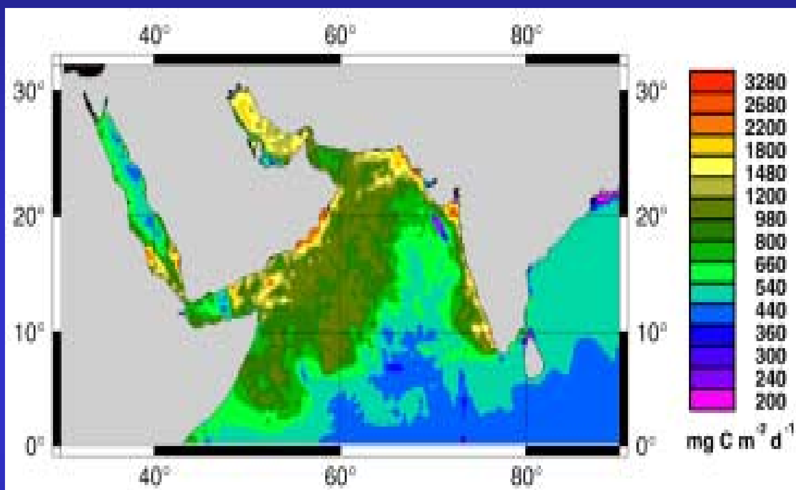
where:

$$\Pi^B_\lambda(z) = \sec\theta_d \int_{400}^{700} \alpha^B(\lambda)\, I_d(z,\lambda,\theta)d\lambda + 1.20 \int_{400}^{700} \alpha^B(\lambda)\, I_s(z,\lambda)\, d\lambda$$

$P_{Z,T}$ computed over: 61 λ's; depth step=0.5m; time interval = 1hour; integration to Zp

LECTURE 3

MODELLING PP ON OCEAN BASIN SCALES



CASE STUDY:

MODELLING PRIMARY AND NEW PRODUCTION IN THE NORTHWEST INDIAN OCEAN REGION

WATTS *et al.* (1999)
Marine Ecology Progress Series Vol 183: 1-12

To recap:

LECTURE 2 - MODELLING PP ON LOCAL SCALES

Two possible procedures for establishing a local algorithm for the calculation of daily PP over the ocean water column, $P_{z,T}$ at a given location.

Based on the following information:

- **Irradiance at sea-surface**
- **Biomass profile**
- **P-I parameters**

LECTURE 3 - MODELLING PP ON OCEAN BASIN SCALES

AIM: To extrapolate the local algorithm to large, horizontal scale

REQUIRES: implementation of the local algorithm at many field points (pixels) in the region of interest

REQUIRES: above information supplied to EVERY PIXEL

HOW DO WE DO THIS?

AIM:

To develop a protocol for the assignment of the biomass profile and P-I parameters to every pixel.

PARTITIONING OF THE REGION OF INTEREST TO FACILITATE LARGE-SCALE CALCULATIONS:

DYNAMIC BIOGEOCHEMICAL PROVINCES

Platt & Sathyendranath (1988)- *Science, 241, 1613-1620;*

Platt *et al.* (1995) *Phil. Trans. R. Soc. Lond.* B (348): 191-202

CONCEPT:

Assumes that in the pelagic ecosystem the rates of important ecophysiological and biogeochemical processes are under physical control (particularly the photosynthetic rate)

We partition the ocean by delineating those areas which have a common physical forcing

DYNAMIC BIOGEOCHEMICAL PROVINCES

IDEA IS STILL YOUNG!

CHARACTERISTICS:

- **THEY ARE FEW IN NUMBER FOR A GIVEN OCEAN (~10 PER OCEAN/50 GLOBALLY)**
- **THEY HAVE DYNAMIC BOUNDARIES SO THAT THE BOUNDARIES CAN MOVE ACCORDING TO SEASON AND BETWEEN YEARS**
- **THESE BOUNDARIES SHOULD BE ACCESSIBLE TO REMOTE SENSING**
- **WITHIN SEASONS EACH PROVINCE SHOULD HAVE PREDICTABLE OR STABLE PROPERTIES (BIOMASS PROFILE AND P-I PARAMETERS)**
- **THEY PROVIDE TEMPLATES FOR THE APPLICATION OF PROVINCE SPECIFIC RULES ie THEY MUST BE ABLE TO INCORPORATE LOCAL KNOWLEDGE**

DYNAMIC BIOGEOCHEMICAL PROVINCES

SOME CASE STUDIES:

LONGHURST *et al.* (1995) - *J Plankton Res* 17(6) 1245-1271

•LONGHURST (1998) - Ecological Geography of the Sea. San Diego: Academic Press

•SATHYENDRANATH *et al* (1995) *DSRI* 42(10): 1773-802

•WATTS *et al* (1999) *MEPS* 183: 1-12

SUMMARY

LECTURE 1:

ACTED AS A BASIS FOR LECTURES 2 & 3:

PROCESSES AND PROPERTIES RELEVANT TO MODELLING PRIMARY PRODUCTION:

- Properties of oceanic phytoplankton
- Phytoplankton biomass: BIOMASS PROFILE
- Primary production: MODEL VALIDATION
- The P-I curve: P-I PARAMETERS
- The photic zone depth: Zp, K

LECTURE 2:

MODELLING PP ON A LOCAL SCALE:

- simple approach: analytic model
- complex approach: numerical model

LECTURE 3:

EXTRAPOLATION OF LOCAL MODEL TO OCEAN BASIN SCALES; BIOGEOCHEMICAL PROVINCES

Demo of software for curve fitting to observational data for retrieval of parameters for input into the models (above)